
L. E. Calvet & A. J. Fisher:

Multifractal Volatility: Theory, Forecasting, and Pricing

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This starkly original book on multifractal volatility brings remarkable advances in volatility modeling with a focus on financial applications. In a limited space of 272 pages, the reader is enjoyably led to explore the unique world of Laurent E. Calvet and Adlai J. Fisher along with their innovative class of models for financial volatility modeling. The content of the chapters relies heavily on their published series of papers in respected academic journals, yet the importance of the work is much easier to see when presented in a coherent book.

Generally, research on financial markets is a highly empirical discipline that has become one of the most active fields in the social sciences. Beyond the motivation of high expected profits, or perhaps the high willingness of market participants to pay for this kind of research, there are deep intellectual reasons driving this huge interest in financial markets. Over the past 30 years, financial econometricians have uncovered fascinating properties and regularities of asset returns. The availability of this high-frequency data brought other very interesting insights. Also, the market itself has been reminding researchers of the crucial properties that help to understand its behavior: events like sudden stock market crashes are a good example. Black Monday, 19 October, 1987, prompted us to drastically take note that the underlying distribution of asset returns is far from normal.

Regardless of the huge amount of commotion present in the data, the most promising part of the research was the predictability of the second distributional moment of returns. Volatility clustering, persistence and its time variation are directly observable even to random bystanders. Thus, a vast literature emerged on the phenomenon, and time-varying volatility approaches became a traditional benchmark. The most direct way to capture the dynamics of second moments is to write a process for the volatility of returns, conditional on past returns and other available information. Robert Engle was the first to propose this approach to volatility modeling by allowing for conditional heteroscedasticity to the autoregressive process of returns (ARCH model). His seminal contribution that changed the world of stock markets modeling was recognized in 2003 by being awarded the Nobel Memorial Prize in Economic Sciences. An alternative approach that has been attracting interest in recent years permits the conditional mean and variance of financial returns to depend on a latent state that can not be observed directly; the so-called regime-switching approach was pioneered by James Hamilton. The technical difficulty of this approach is that in order to capture the complexity of the asset markets one would like to allow for many possible states in volatility resulting in an inapplicable model with a very large number of parameters to be estimated.

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In this book, the authors break the barrier and bring a completely fresh, unified and complete approach to volatility modeling. During their graduate years at Yale under the supervision of Sterling Professor, Benoît Mandelbrot, the father of fractal geometry, they developed the original Multifractal Model of Asset Returns (MMAR). In order to allow for volatility forecasting and asset pricing, they independently developed a Markov model with multifrequency characteristics, and then the Markov-Switching Multifractal (MSM) a few years later. MSM assumes the volatility to be the product of a large number of discrete variables. Each of these variables are allowed to randomly switch to a new value drawn from a common distribution. The approach is thus based on pure regime-switching at all frequencies.

To understand the concept of multifractals, the study of fractals is a good starting point. A fractal, or self-similar process, is built on the concept of scale invariance. In 1963, Benoît Mandelbrot suggested that the shape of the distribution of returns should be invariant when time scale is changed. Generally, he viewed fractals as *“a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole”* (Benoît Mandelbrot). Many popular volatility models belonging mainly to the family of fractional integrated processes (i.e. FIGARCH) capture this feature of the time series, sometimes called long memory. When a process follows one of the integrated models, we also call it unifractal. Multifractal theory generalizes and extends these ideas. While fractals are characterized primarily by a single number—fractal dimension—multifractals are characterized by a func-

tion. In a very simplified way, the multifractal process can be understood as a process of having a property of scaling in q th moments. Applying the concept to finance, the authors propose that the q th moment of absolute returns scales as a power function of the frequency of observation, provided that it exists. A rigorous understanding of multifractal theory is expected of the reader in order to be familiar with several advanced mathematical concepts, but very basic intuition can be gained from this line of explanation.

The development of a multifractal approach to financial returns modeling is the main theme of this book. It does a good job in describing the methodology in full detail including the statistical inference for estimation, empirical examples and a comparison of out-of-sample forecasts with other well-known models. The authors also provide evidence that their approach outperforms the fractional models (i.e. FIGARCH) used to model the long memory. This suggests that unifractal models may be insufficient to explain the complex stock market returns behavior. The authors also provide evidence that their model can capture the well-known stylized facts of volatility; long-range (as well as short-range) dependence in volatility, fat tails of return distribution, and also volatility jumps. Moreover, the authors provide a multivariate setting for modeling co-movement and compare it to the traditional constant correlation CC-GARCH of Bollerslev. The theory is also complete for the multivariate part and includes the statistical inference for the estimation of the model. Finally, the authors explore the implications of multifrequency risk for the valuation of assets by assuming that the fundamentals are subject to multifrequency risk given

the heterogeneity of the news that drives the financial returns. The appeal of the multifractal approach becomes clear when one considers its various applications as a whole. The Markov-switching approach to multifractal modeling provides a parsimonious and rigorous way of modeling a financial time series. It is indeed a very nice way to connect phenomena that have only been modeled separately in the past.

It is rare for a new book to receive so much advance praise from so many lead-

ing researchers, and even more striking is the fact that the field of multifractal processes is far from the mainstream of financial econometrics. The authors did an outstanding job in putting the material for this book together and bringing the financial world new and very promising perspectives on asset prices modeling. Since the content of the book is extremely useful to a general audience interested in the latest advances in volatility modeling, I would highly recommend it as a *must-read*.