

Nonlinear Inflation Expectations and Endogenous Fluctuations

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Received 25 February 2009; Accepted 9 March 2010

Abstract The standard new Keynesian monetary policy problem is presentable as a set of linearized equations, for values of endogenous variables relatively close to their steady-state. As a result, only three possibilities are admissible in terms of long-term dynamics: the equilibrium may be a stable node, an unstable node or a saddle point. Fixed point stability (a stable node) is generally guaranteed for an active monetary policy rule. The benchmark model also considers extremely simple assumptions about expectations (perfect foresight is frequently assumed). In this paper, one inquires how a change in the way inflation expectations are modelled implies a change in monetary policy results, when an active Taylor rule is considered. By assuming that inflation expectations are constrained by the evolution of the output gap, we radically modify the implications of policy intervention: endogenous cycles, of various periodicities, and chaotic motion will be observable for reasonable parameter values.

Keywords Monetary policy, Taylor rule, inflation expectations, endogenous business cycles, nonlinear dynamics and chaos

JEL classification E52, E32, C61

1. Introduction

The success of monetary policy intervention in controlling inflation in most of the developed world along the past few decades is the result, among other factors, of the change in the theoretical paradigm followed in macroeconomic science. Since the famous analysis about the inconsistency problem (Kydlund and Prescott 1977; Barro and Gordon 1983), it is widely accepted that the main goal of monetary policy should consist in fighting price instability, rather than worrying about real stabilization. This idea became clearer with the development of the model that has gained the central position in the explanation of central banks behaviour: the new Keynesian monetary policy problem (see, among many others, Goodfriend and King 1997; Clarida et al. 1999; Woodford 2003).

The benchmark new Keynesian model has been built over the staggered price-setting analysis of Calvo (1983), which has allowed recovering the Phillips curve relation. It is admissible to establish a relation between the contemporaneous values of the inflation rate and of the output gap, through a parameter that reflects the degree of price stickiness; when this relation is augmented by a term that relates the present period's

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inflation with expectations about future inflation, we can establish the central piece of the new monetary policy paradigm, which is the ‘new Keynesian Phillips curve’ (this denomination was initially proposed by Roberts 1995).

Alongside with the aggregate supply relation that the Phillips curve defines, another state constraint is essential to describe the short-run environment in which monetary authorities are compelled to take decisions; this is an IS equation that characterizes how the real economic activity responds to changes in the real interest rate.

With the knowledge of the previous two state equations, the central bank has a problem to solve, which is to maintain price stability and, if possible, to guarantee some positive difference between effective output and its potential level (if this does not hurt the inflation objective). The most immediate solution for this problem would be to consider an optimal control setup, under which the central bank minimizes the distance between the observed inflation rate and output gap relatively to the corresponding target values it defines. The constraints of this intertemporal problem are the Phillips curve and the IS equations. The control variable is the nominal interest rate, i.e., the monetary authority chooses the time path of the interest rate that optimizes its utility function over time.

If one considers the benchmark version of the optimizing model, a problem arises: the optimal interest rate path does not correspond to a stable path, and therefore the intended long term optimal values of inflation and output are not accomplished. In this sense, the stability of the equilibrium becomes a central issue in the way monetary policy is conducted. If optimal policy is not stable, it is necessary to find a less than optimal result that guarantees stability. This is generally assured by assuming an ad-hoc interest rate rule instead of following the optimal path.

The influential work of Taylor (1993) and the huge amount of literature that it has originated seems to give a satisfactory answer to the stability concern (see, among many others, McCallum and Nelson 1999; Benhabib et al. 2001; Svensson and Woodford 2003; Benigno and Woodford 2005). It has become widely accepted that an active Taylor rule (i.e., a monetary policy rule under which in response to an increase in inflation the central bank raises the nominal interest rate by more than the increase in inflation), has stabilizing effects. Intuitively, this appears correct: inflationary pressures are fought by a monetary policy that triggers an increase in the real interest rate, which should have the effect of slowing down aggregate demand and, therefore, sustain the rise in the general price level.

The described monetary policy problem is essentially linear.¹ Replacing in the IS curve the nominal interest rate by a rule in which this rate is dependent on inflation (and also on the output gap), the reduced form of the problem will be a system of two difference equations where, under perfect foresight, the output gap and the inflation rate depend linearly on previous period values of these two variables (and, also lin-

¹ To be precise, the new Keynesian model is founded on the optimizing behaviour of firms and households, which gives place to a setting where log-linearization is required in order to be possible to analyze the relation between economic aggregates. Thus, both the linear dynamic IS and the linear Phillips curve should be understood as the outcome of a steady-state vicinity evaluation. Over these relations we will apply a departure from rational expectations that introduces a nonlinearity that holds even in the neighbourhood of the long-term equilibrium position.

early, on eventual stochastic shocks on demand and supply). When changing the linear form of the model the stability result can give place to endogenous cycles, which essentially mean that a public policy oriented to attain price stability may not achieve a full stability result, but it can produce fluctuations, that will be more or less predictable depending on their own periodicity.

Concerning the introduction of nonlinearities, authors follow essentially two paths:

(i) When assuming the optimal problem, the original framework considers a quadratic objective function. Various authors, like Cukierman (1999), Ruge-Murcia (2002, 2004), Nobay and Peel (2003), Dolado et al. (2004) and Surico (2004), claim that a symmetric objective function does not represent properly the true policy problem (authorities do not perceive as equally important positive and negative deviations from the target values of inflation and output gap). Thus, nonlinearities and the possibility of long term endogenous fluctuations arise in a way that is consistent with empirical evidence.

(ii) Also consistent with empirical evidence is the fact that the Phillips curve can hardly be modelled through a linear relation. Clark et al. (1996), Debelle and Laxton (1997), Schalling (1999), Tambakis (1999) and Akerlof et al. (2001), among others, present evidence and argue against a linear relation between the inflation rate and the output gap, in the short-run. Gomes et al. (2007) prove that for a specific functional form of a non linear Phillips curve, endogenous cycles are found, and this corresponds mainly to cases in which no identifiable periodicity is encountered (i.e., when assuming a non linear Phillips curve chaotic motion can be generated for values of parameters that do not depart significantly from empirical data).

In this paper, we consider the non optimal monetary policy model (i.e., we assume a Taylor rule) and linear Phillips and IS equations that are linear in the relation between contemporaneous values. The nonlinearity is introduced by departing from the perfect foresight assumption regarding inflation expectations. This is also a subject debated in the literature, for instance by Jensen (2005), who considers that policy affects expectations about future policy. In Branch and McGough (2009) and Gomes (2006), inflation expectations are modified by considering heterogeneous agents, who predict future inflation in different ways; under bounded rationality (i.e., under a discrete choice mechanism for the switching between expectation rules) chaotic motion is also identified in this case.

In the present setting, we depart from perfect foresight by assuming that agents will form expectations about inflation having in consideration the output gap. The rule is as follows: when the output gap is equal to its target value, as defined by the central bank and perceived by private agents, the perfect foresight will hold; if the output gap rises above that benchmark value, then the expected inflation will also rise above the perfect foresight value; if the output gap falls below the target, agents will predict an inflation value below the perfect foresight value; finally, for strong recessions (output gap clearly negative), agents expect inflation to rise faster (that is, strong recessions will be a symptom of an economy where institutions do not work, and therefore the control of price stability does not function properly).²

² A similar expectations formation rule is considered in Gomes et al. (2008); the present paper extends such

This simple assumption over the original monetary policy problem imposes relevant changes on the dynamic behaviour of variables, namely chaos and cycles of various periodicities are obtained. Therefore, one concludes that monetary policy (under an active interest rate rule) does not yield necessarily a fixed point result, but cycles of several periodicities are observable, when considering parameter values that intuitively are reasonable.

The remainder of the paper is organized as follows. Section 2 discusses the intuition behind the inflation expectations rule; Section 3 presents the analytical structure of the model; Section 4 characterizes global dynamics; in Section 5, growth issues are addressed; and, finally, Section 6 concludes.

2. Inflation expectations

The simplest approach to modelling expectations consists in assuming perfect foresight. Under perfect foresight, agents have a complete knowledge about the economy. They know how every other agent will act and how monetary authorities will conduct their policy. In turn, authorities should also understand without doubts the decisions that the private economy take in every moment of time, being as well able to predict and anticipate the decisions of all economic agents. This implies a world where agents' choices become the best response to the choices of third parties.

The analysis will be pursued under a deterministic market environment and, thus, stochastic disturbances will be overlooked. This does not mean that exogenous fluctuations are irrelevant; there will certainly be a large set of external influences determining the relation between the inflation rate and the output gap. However, by ignoring them one can concentrate on the nonlinear relations that will emerge in the proposed setting; without external shocks, all the obtained fluctuations are necessarily endogenous, i.e., the result of how the variables in the model are related. Note, as well, that the notion of perfect foresight is the deterministic equivalent of rational expectations: under rational expectations, random changes in the values of variables are admissible, but these changes correspond to uncorrelated errors, i.e., agents possibly make mistakes but the mistakes are uncorrelated. In this way, ignoring forecast errors has no decisive impact on the conclusions one wants to withdraw because in the absence of systematic deviations from the fundamental solution, the considered errors are innocuous in terms of the obtained long-term outcome.

Nevertheless, one should take into account that the perfect foresight/rational expectations approach to expectations formation is, in most cases, excessively narrow, implying full information and full efficiency in the use of information - it must be implicitly assumed that the average economic agent has the ability to instantly acquire knowledge, at every time moment, about the 'true' model of the economy, a capability that the economic science avoids to attribute even to professional economists, who have to estimate parameters, over time, in order to effectively understand the relations

analysis, exploring new dynamic results and developing further the implications of the obtained nonlinear results. Furthermore, in this paper we concentrate on inflation dynamics; therefore, the output gap is kept at its perfect foresight level. In the cited paper, bounded rationality is pervasive: it respects not only to inflation but also to the output gap.

that are established among economic aggregates. This is why macroeconomics has become increasingly concerned with alternative methods of modelling expectations.

The most widely approached alternative to rational expectations relates to the consideration of learning mechanisms (see, e.g., Evans and Honkapohja 2001). Under learning, agents will predict future outcomes by collecting and processing information over time; as time goes by, the set of available information increases and the predictability capabilities will also increase. Under this interpretation, rational expectations can be seen as the possible asymptotic state to which the economy converges: one can conceive a long-run scenario when all the relevant information about some economic subject has been gathered and processed, such that the individual agent no longer makes mistakes. This view indicates that perfect foresight should not be interpreted as an automatic capability of economic agents; however, it can correspond to the long-run outcome that will dominate when the learning process is complete and the agent acquires a perfect ability to forecast the future (once the agent has learned all that there is to learn, then no more systematic errors will be made).

In the present analysis we ignore any kind of learning mechanism and stick with the perfect foresight assumption that gives place to the fixed point outcome in the presence of an active monetary policy rule; nevertheless, over perfect foresight, one takes an additional assumption that reflects how the private economy responds, in terms of the way it perceives price evolution, to fluctuations in output.

We assume that output gap expectations are solely the outcome of a perfect foresight evaluation: $E_t x_{t+1} = x_{t+1}$. The output gap variable is defined as the difference between effective output and potential output (in logs), that is, $x_t = \ln y_t - \ln \hat{y}_t$. Relatively to the inflation expectations, the perfect foresight prediction is adjusted by a term that translates the way individuals think the difference between effective and potential output will affect the rise in prices. Thus, we consider $E_t \pi_{t+1} = \pi_{t+1} \cdot \xi(x_t)$.

Function $\xi(x_t)$ must be such that when the output gap is equal to some predefined value (that here we consider to be the target value of the central bank for this variable: x^*), the value of this function is 1, that is, perfect foresight holds. When $x_t > x^*$, the output gap has assumed a value above 'normal', and thus agents will suspect that only a rise in prices will be able to maintain such abnormally high output gap, and hence they will expect prices to rise above the perfect foresight value. If $x_t < x^*$, private agents will perceive a slowdown of the economic activity, and therefore they introduce a penalty term in their predictions, which means that the expected inflation value will remain below the benchmark value.

Finally, when the output gap becomes extremely low relatively to the corresponding target value, this will be understood as a serious problem of economic malfunctioning, probably associated to an inability of the institutions to fulfil their regulatory role, and therefore very low levels (in principle, negative) of the output gap will be understood as eventually producing a faster rise in prices because the monetary authority becomes unable of controlling the production of money and the interest rates.

Figure 1 presents the shape of function $\xi(x_t)$, when this obeys to the characteristics described above. Parameter $\sigma > 0$ is defined in order to present the location of the point in which the function reaches a minimum and therefore the expected inflation

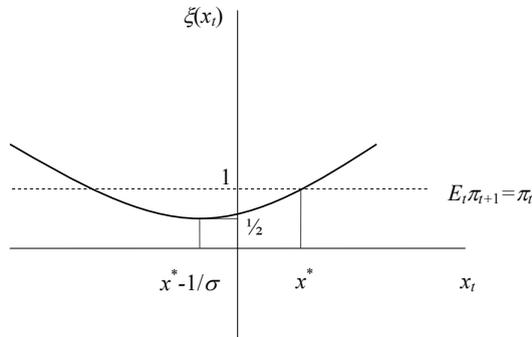


Figure 1. Function $\xi(x_t)$

is the lowest relatively to the perfect foresight value. Note, as stated, that three areas are identifiable: high inflation is expected in periods of expansion or strong recession; moderate recession implies low expected inflation.

The function in Figure 1 can be translated analytically as follows:

$$\xi(x_t) = 1 + \sigma \cdot (x_t - x^*) + \frac{\sigma^2}{2} \cdot (x_t - x^*)^2.$$

Synthesizing, over the original new Keynesian monetary model one introduces only one modification: we bend the line $E_t \pi_{t+1} = \pi_{t+1}$ in order to illustrate how individuals and firms react (in terms of price evolution predictions) to the output gap departures from a reference value.

In the proposed setting, private agents intend to solve their optimization problems as if perfect foresight continued to hold; however, the available set of information and the way they interpret reality makes it hard to avoid systematic mistakes. The accuracy of predictions is progressively lost as the economy departs from the 'reference' state, with this reference corresponding to the optimal equilibrium (which corresponds to the goal that the central bank has defined for the real efficiency of the economy). Hence, instead of assuming perfect foresight, we can interpret the behaviour of the agents as being 'near rational'. The notion of near rationality and the possibility of small deviations from rationality implying relevant departures from expected economic outcomes have been thoroughly discussed in the economic literature, for instance by Akerlof and Yellen (1985), Haltiwanger and Waldman (1989), Bomfim and Diebold (1997) and Weder (2004).

Next section incorporates the near rationality assumption we have been exploring into the monetary policy framework.

3. The monetary policy model

In what follows we describe the main features of the conventional new Keynesian monetary model. The state constraints are, on the demand side, a dynamic IS equation,

and, as an aggregate supply relation, a new Keynesian Phillips curve. The first relates the output gap to the expected real interest rate,

$$x_t = -\varphi \cdot (i_t - E_t \pi_{t+1}) + E_t x_{t+1} + g_t, \quad x_0 \text{ is given.} \quad (1)$$

Parameter $\varphi > 0$ is the output gap - interest rate elasticity and variable i_t defines the nominal interest rate. Variable g_t corresponds to a demand stochastic component and it is defined through an autoregressive Markov process, $g_t = \mu g_{t-1} + \hat{g}_t$, $0 \leq \mu \leq 1$, $\hat{g}_t \sim iid(0, \sigma_g^2)$. Subsequently, we ignore the stochastic component of the equation in order to highlight the presence of endogenous fluctuations. The only consequence of neglecting the stochastic process is that we will be able to guarantee that the fluctuations to be observed are purely deterministic; if we leaved this component in the specification of the model, we would have two simultaneous sources of cyclical motion: the one that is generated inside the model by the nonlinear relation among variables and the one that respects to other events on the economy that are not subject to analysis in the current study (in practice, this is what variable g_t represents: all the factors, on the demand side, that can influence the relation between output and inflation that are overlooked by the considered analysis).

On the supply side, the Phillips curve relates contemporaneous inflation to the output gap and to the next period inflation expectations,

$$\pi_t = \lambda x_t + \beta \cdot E_t \pi_{t+1} + u_t, \quad \pi_0 \text{ is given.} \quad (2)$$

Parameter $\lambda \in (0, 1)$ defines the degree of price flexibility/stickiness, that is, it is an inflation-output elasticity. The higher the value of this parameter the lower will be the degree of price stickiness or rigidity. Parameter $\beta < 1$ is an intertemporal discount factor, and variable u_t translates a supply stochastic component, that reflects possible cost push shocks. As in the demand case, an autoregressive process is assumed: $u_t = \rho u_{t-1} + \hat{u}_t$, $0 \leq \rho \leq 1$, $\hat{u}_t \sim iid(0, \sigma_u^2)$; also as in the demand case, this term is ignored under the discussion of endogenous fluctuations.

To complete the model, one takes a conventional Taylor rule, which is given by the following expression (a similar Taylor rule can be found in Clarida et al. 1999),

$$i_t = i^* + \gamma_\pi \cdot (E_t \pi_{t+1} - \pi^*) + \gamma_x \cdot x_t, \quad (3)$$

where i^* defines the equilibrium nominal interest rate, π^* is the inflation target that the central bank sets (low, but positive in order to guarantee relative price variations without the need of nominal decreasing of prices and wages), and γ_π and γ_x are the policy parameters that reflect how the central bank reacts in terms of interest rate changes, when economic conditions provoke variations in the values of inflation and effective output.

As stated in the introduction, an active interest rate rule is, normally, stabilizing, meaning that stability is attained when there is an interest rate response to inflation changes that are stronger than a one-to-one change; this implies imposing the constraint $\gamma_\pi > 1$.

Replacing the Taylor rule (3) in the IS expression (1), and assuming perfect foresight for the output gap, we get the following relation between output gap and inflation rate, regardless from the expectations about inflation,

$$x_{t+1} = \varphi \cdot (i^* - \gamma_\pi \pi^*) + [1 + \varphi \gamma_x - \varphi(\gamma_\pi - 1) \cdot \lambda / \beta] \cdot x_t + [\varphi(\gamma_\pi - 1) / \beta] \cdot \pi_t. \quad (4)$$

The Phillips curve can be rewritten, having in consideration the way we have defined inflation expectations in the previous section,

$$\pi_{t+1} = (1/\beta) \cdot [\pi_t / \xi(x_t)] - (\lambda/\beta) \cdot [x_t / \xi(x_t)]. \quad (5)$$

The system one wants to analyze is the difference equations system (4)–(5). This is the conventional problem for $\sigma = 0$, and it departs from this case as we increase the value of the parameter (the higher the value of σ , the more the inflation expectations rule ‘bends’ relatively to the perfect foresight case). Except in the known particular case $\sigma = 0$, the analysis of the steady state and of local dynamics becomes difficult. Solving for the steady state one would obtain multiple equilibria (a third order polynomial would be obtained and thus three equilibrium points would arise); nevertheless, the combinations of parameters that define the steady state points are cumbersome and it becomes difficult to extract some meaningful information from them. Without the steady state values, local dynamic analysis is not feasible as well. The next section concentrates on a global analysis of the underlying dynamics, which is essentially a numerical and graphical analysis.

4. Global dynamics

System (4)–(5) involves a linear equation and a nonlinear equation. As we will understand below, the presence of this nonlinear relation opens the possibility for finding strange dynamics characterizing the long-term behaviour of endogenous variables.

Otherwise stated, the following parameter values are considered: $\beta = 0.96$; $\gamma_x = 0.5$; $\gamma_\pi = 2.2$; $\sigma = 25$; $\varphi = 0.01$; $\lambda = 0.75$; $\pi^* = 0.02$; $x^* = 0.03$; $i^* = 0.01$. Note that for reasonable initial values of variables inflation and output gap, we find no limit for the basin of attraction, and therefore any economically meaningful initial values can be considered for the matter at hand.

We begin by presenting some bifurcation diagrams.³ Figures 2 and 3 display the long term possible outcomes of the output gap and the inflation rate for different values of the parameter that defines the nature of the monetary policy $\gamma_\pi < 1$ respects to a passive monetary policy and $\gamma_\pi > 1$ to an active policy. The most striking and important evidence in these figures is that instability prevails for a passive interest rate rule, that is, when the central bank responds to the rise of inflation with a less than one-to-one variation in the nominal interest rate. Instability is characterized in this case by a divergence of the output gap to infinity and of the inflation rate to zero.

³ The various figures presented in this section are drawn using IDMC software (Interactive Dynamical Model Calculator). This is a free software program available at www.dss.uniud.it/nonlinear, and copyright of Marji Lines and Alfredo Medio.

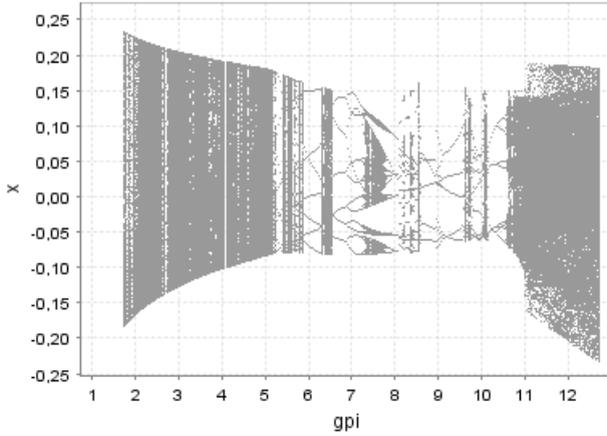


Figure 2. Bifurcation diagram γ_π, x_t

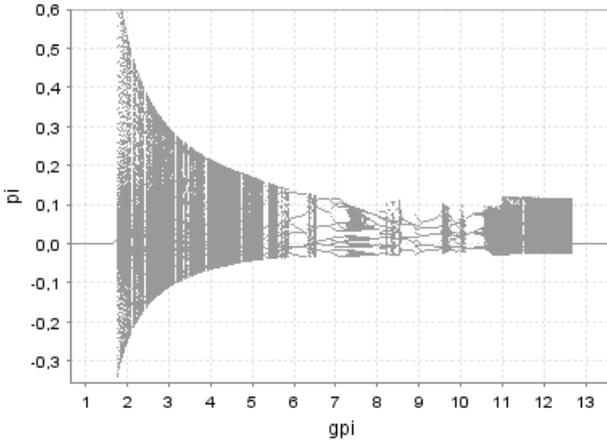


Figure 3. Bifurcation diagram γ_π, π_t

When the policy parameter assumes a value higher than 1, we observe that instability continues to prevail as long as $\gamma_\rho i$ is kept below 1.725; after this value, the modified nonlinear expectations model implies the presence of cycles of multiple orders until an extremely large value of the parameter is considered. Thus, a strongly aggressive monetary policy (but not exaggeratedly strong) is the only escape, in the considered framework, relatively to pure instability. As long as the policy parameter $\gamma_\rho i$ remains inside the interval (1.725; 12.75), given the other assumed parameter values, we will observe the presence of bounded instability, i.e., the output gap and the inflation rate

will fluctuate, but inside a given set of boundaries. These fluctuations are, for some values of the parameter, completely irregular (the system is characterized by the presence of a-periodic and chaotic motion), while for some values of the policy parameter in the mentioned interval, periodic cycles, of many possible periodicities are displayed.

Basically, we note that some regions in Figures 2 and 3 define cases in which low periodicity cycles exist, while in other areas of the graphics it is evident the presence of chaos: the variable can assume practically any value on a given interval. The existence of chaos can be confirmed under one of the most striking features of chaotic systems: the presence of sensitive dependence on initial conditions (SDIC); SDIC indicates that if a same deterministic system is initiated in two different states, even if very close to each other, then the corresponding trajectories will follow completely uncorrelated paths. This occurs for many of the referred potential values of the policy parameter; the time series that we will present below reflect this outcome.

We will highlight further the presence of endogenous fluctuations in the figures that follow; nevertheless just by looking to the bifurcation diagrams (that are drawn for the 1,000 observations after the first 1,000 transient ones) it is evident that a-periodicity is present.

One can explore as well the presence of cycles of different orders in the space of parameters. With Figures 4 to 6, we are able to observe that all sorts of periodicities are obtainable for different values of parameters. Regions in white contain the possibility of chaotic motion (more specifically, they correspond to areas where no cycle of periodicity lower than 20 is observable). These figures reveal that the dynamic system is deeply sensitive to small changes in most of the parameter values. The referred graphics relate the policy parameter γ_π to three other relevant parameters: the degree in which expectations depart from perfect foresight, σ , the parameter that measures the degree of price stickiness, λ , and the second policy parameter, γ_x . In each case,

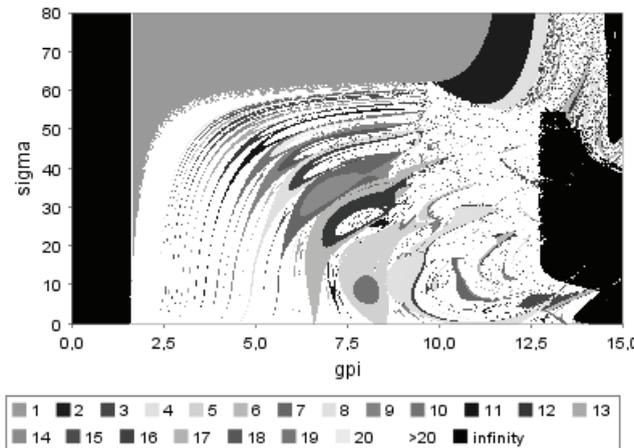


Figure 4. Cycles in the space of parameters (γ_π, σ)

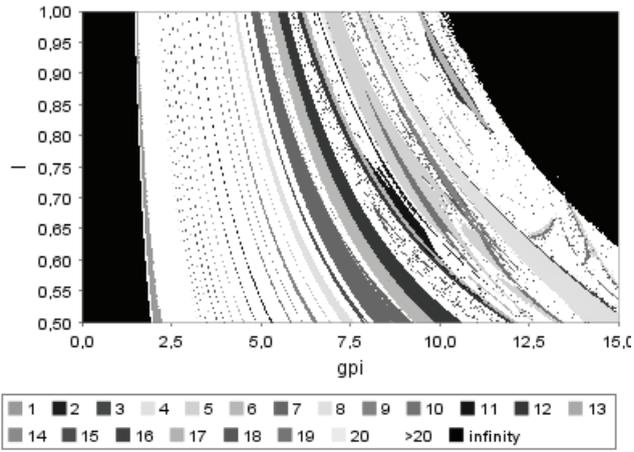


Figure 5. Cycles in the space of parameters (γ_π, λ)

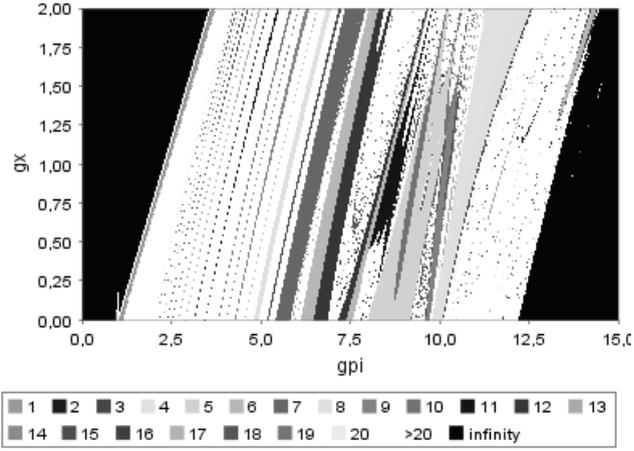


Figure 6. Cycles in the space of parameters (γ_π, γ_x)

cycles of multiple periodicities are depicted.

In Figure 4, we confirm that only for $\gamma_\pi > 1.725$, instability can be ruled out; this is true for any value of σ . For very large values of this parameter, a fixed-point outcome is attainable; however, the most frequent result consists in observing cycles of very large periodicity or complete a-periodicity. Figures 5 and 6 indicate that the other two parameters selected for the analysis (λ and γ_x) are not decisive in terms of dynamic behaviour; cycles will be observed regardless of the values assumed by any of these two constants. Nevertheless, Figure 5 points to the fact that stickier prices (lower λ)

increase the range of values of γ_π that allow for fluctuations.

We now take the set of parameter values defined in the beginning of this section to display a few attractors, i.e., the long term relation between our two endogenous variables. Figure 7 considers precisely the initial set of values. To understand how the dynamics can be modified, we vary some of the parameter values to present the graphics in Figures 8 and 9. All the attractors are drawn with 100,000 iterations after excluding the first 1,000 transients.

In Figure 8, we significantly change the value of parameter γ_π in order to place the

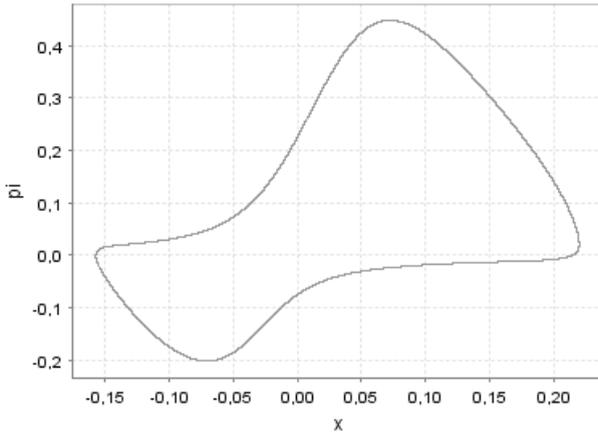


Figure 7. Attractor (x_t, π_t)

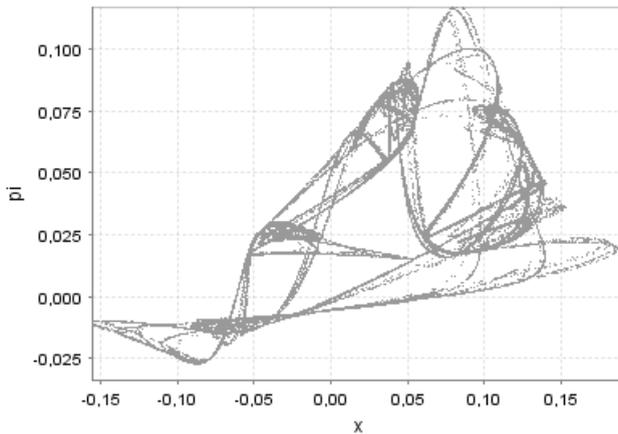


Figure 8. Attractor (x_t, π_t) , $\gamma_\pi = 11$

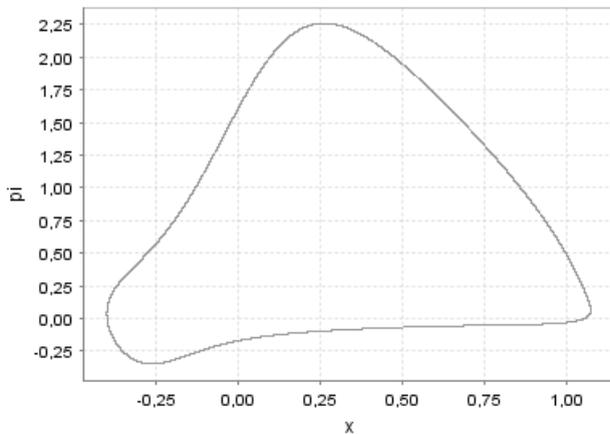


Figure 9. Attractor (x_t, π_t) , $\sigma = 5$

system at a point in which it is clear the evidence of chaos; while in Figure 7 there is quasi-periodicity (i.e., SDIC is observable, but some regularity is found on the long-term relation between the output gap and the inflation rate), in the second case, we are in the presence of a strange attractor; the relation between the two variables does not obey to any kind of regular behaviour. In Figure 9, we return to the original value of the Taylor rule’s policy parameter and we considerably decrease the value of σ ; there are no qualitative changes, in the sense that the attractor continues to correspond to a quasi-periodic relation between the two endogenous variables.

Finally, a pair of long-term time series is presented, in order to illustrate that the endogenous cycles involve some degree of persistence. The values of parameters used to

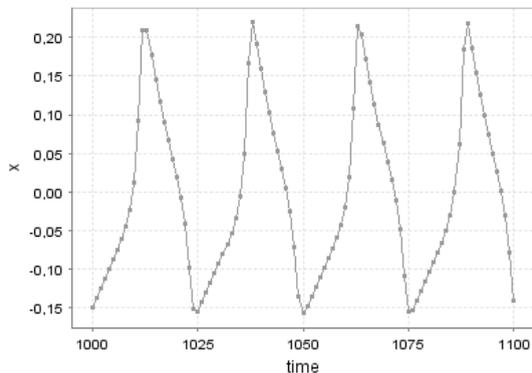


Figure 10. Time series x_t

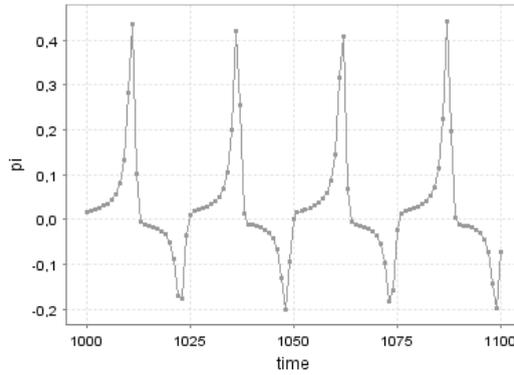


Figure 11. Time series π_t

draw these series are the ones in the benchmark case. Note that both variables assume positive and negative values, that is, periods of inflation and deflation are observed, as well as periods when the effective output is above and below the potential level (Figures 10 and 11).

In what concerns the output gap time series, notice that falls in the output gap occur at a relatively constant pace, but recoveries follow a different pattern: they begin at a slow rate, but then they accelerate when the output gap becomes positive. In both cases (decreasing and increasing output gap phases), the time length for getting from the highest/lowest output gap to the other extreme is around 11 to 12 periods. Regarding inflation, we observe that both inflationary and deflationary processes begin slowly but tend to accelerate to a given peak; once this peak is reached, the inflation/deflation situation is rapidly inverted. Note, as well, that episodes of inflation are more intense than the ones concerning deflation (in the first case, the inflation rate will reach higher values, in absolute value).

5. Growth implications

Monetary policy analysis is undertaken through two state equations that define short-run economic conditions. These can be integrated with a long term growth analysis. Growth models are generally developed under a competitive framework and they are specially designed to analyze the trend of growth, i.e., they are built in order to characterize potential output motion. Consider a capital accumulation equation

$$k_{t+1} = Ak_t^\alpha - c_t + (1 - \delta) \cdot k_t, \quad k_0 \text{ is given.} \tag{6}$$

In this equation, k_t and c_t represent, respectively, per capita physical capital and consumption. Parameter $A > 0$ is a technological index, $\alpha \in (0, 1)$ respects to the output-physical capital elasticity and δ defines a positive depreciation rate. From growth literature it is well known that, given a representative agent that maximizes an intertem-

poral flow of consumption utility functions, the growth problem is reduced to a two equations system describing the motion in time of the consumption and of the capital variable. Then, the long term behaviour of output can be withdrawn from the production function, once we know how the rule of capital accumulation and the optimization behaviour of the representative consumer imply a given path for the capital stock.

Therefore, we can use the growth problem to get to the potential level of output, $\hat{y}_t = Ak_t^\alpha$. In the real world, we are not concerned with how much it is possible to produce, but how much it is effectively produced. Given the proposed notion of output gap, effective output comes: $y_t = \hat{y}_t e^{x_t}$. In terms of growth rates, $\frac{y_{t+1}}{y_t} - 1 = \frac{\hat{y}_{t+1} e^{x_{t+1}}}{\hat{y}_t e^{x_t}} - 1$.

If the competitive growth model is stable, and neoclassical features define it (i.e., output does not grow in the steady state due to endogenous forces) this means that in the long run we find a fixed point stable result for the potential output, and thus $\hat{y}_{t+1} = \hat{y}_t$; the growth rate of effective output becomes, then, $\frac{y_{t+1}}{y_t} - 1 = \frac{e^{x_{t+1}}}{e^{x_t}} - 1$, that is, the growth rate of effective output depends solely on the growth rate of the output gap. If, instead of neoclassical growth, we take the assumption that the growth model is endogenous (a positive constant growth rate defines the steady state), then potential output grows at a given rate γ , meaning that $\hat{y}_{t+1} = (1 + \gamma)\hat{y}_t$. Also in the case of endogenous growth, one can present effective output growth as a function of the output gap, as follows: $\frac{y_{t+1}}{y_t} - 1 = \frac{(1+\gamma) \cdot e^{x_{t+1}}}{e^{x_t}} - 1$.

The previous reasoning intends to conciliate growth analysis, that under market clearing conditions clearly aims at explaining growth tendencies, with the short run analysis provided by the monetary policy problem: because nominal and real economic conditions are jointly determined in the short term, and since expectations are not necessarily the simple result of a perfect foresight evaluation, then fluctuations can be explained in this policy framework and later added to the growth setup. In this way, we strongly emphasize the idea that business cycles are a short run phenomenon that influences the shape of effective growth in time.

To finish, we present a simple graphical example, taking the benchmark numerical

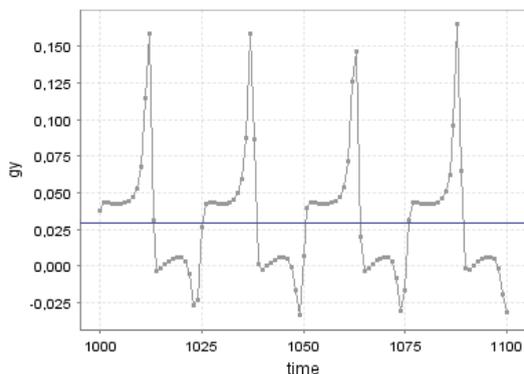


Figure 12. Time series of the growth rate of the effective output ($\gamma = 0.03$)

values of the previous section. For those values, one has concluded that endogenous irregular cycles were present. Now, consider that the potential growth rate (derived from a growth/capital accumulation setup) is, e.g., 3% ($\gamma = 0.03$). Using the definition of effective output derived above, and taking the time series of the output gap in Figure 10, we display in Figure 12 the long term time series of the growth rate of the effective output variable: the growth rate gravitates around the potential value, but since the output gap is not constant, then the evolution of the effective output is subject to endogenous fluctuations.

6. Conclusions

The new Keynesian monetary policy model has two fundamental features: it establishes aggregate demand and aggregate supply relations that are dynamic and subject to the influence of expectations about next period values for real and nominal variables (these relations are derived from well structured micro foundations); and it introduces the relevant role of authorities in choosing the path of the nominal interest rate that best serves the purpose of guaranteeing price stability. It is important to keep in mind that price stability is not necessarily guaranteed by solving an intertemporal optimal control problem, because this can guarantee a steady state that is close to the target defined by the central bank, but that can eventually never be reached given the stability properties of the underlying difference equations system.

Therefore, the model under consideration constitutes not only a good description of private economic behaviour in the short run, but it is also a relevant tool for policy analysis and intervention. The model can be presented in multiple forms, and slightly modified in many ways. Recent literature has proved that slight changes in the benchmark presentation can lead to significant changes in the underlying dynamics, what modifies as well the policy implications one is able to withdraw. In the present paper we have tried to include an additional change relatively to the original model—the idea was essentially to assume that agents do not forecast inflation in a perfect way; even if they possess all the necessary information to decide, they will adjust expectations about inflation to the moment of the business cycle we are in: periods of expansion are understood as periods where inflation will rise faster, while moderate periods of recession imply a feeling that inflation will fall. Thus, we can attach to private economy agents the notion of near rational decisions.

This change in the model's structure introduces significant changes into the dynamics. The model gains a nonlinear character, and as a result we find cycles and chaos for different values of parameters, that replace the unique fixed point result that the original model is able to reproduce. The implications are many: first, monetary policy, that is, the choice of a nominal interest rate rule, no longer gives an absolutely predictable long-term outcome; second, price stability will depend on the degree on which private agents are influenced by output gap changes when formulating expectations; third, it is the short-run relation between nominal and real variables that induces cycles and not the process of capital accumulation, from which one can only withdraw a constant trend of growth.

Acknowledgment Financial support from the Science and Technology Foundation, Lisbon, is gratefully acknowledged, under the contract No. POCTI/ECO/48628/2002, partially funded by the European Regional Development Fund (ERDF). I also thank for the helpful comments of two anonymous referees; the usual disclaimer applies.

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