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More Borda Count Variations for Project Assesment

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Abstract We introduce and analyze the following variants of the Borda rule: median Borda rule, geometric Borda rule, Litvak's method as well as methods based on forming linear combinations of entries in the preference outranking matrix. The properties we focus upon are the elimination of the Condorcet loser as well as several consistency-type criteria.

Keywords Borda rule, median rule, Nash welfare function, outranking matrix, maximin rule, consistency
JEL classification C71, D63, D74

1. Introduction

In a paper delivered in the French Academy of Sciences in 1770 Jean-Charles de Borda introduced a point voting system, nowadays known as the Borda Count (BC, for brevity), for electing best candidates in multi-member voting bodies (Borda's memoir has been translated into English and reprinted in Black (1958) as well as McLean and Urken (1995)). Borda's proposal was specifically designed to replace the then (and now) widespread plurality rule which gives each voter one vote and elects the candidate with larger number of votes than any other candidate.

Despite its initial success in the French Academy, BC has not been widely used in elections involving candidates. Some critics have pointed to BC's vulnerability to strategic voting especially under circumstances where voter groups have information about the popularity of various candidates. Others have taken issue with BC's failure to guarantee the election of an eventual Condorcet winner, i.e. a candidate who is not defeated by any other candidate in pairwise comparisons. In more recent times, the significance of the latter failure has been called into question (Saari, 1995).

Our focus is on some variations of BC in settings involving decision making by expert groups. The setting is admittedly a special one and ignores the wider context in which most group decisions are made. So, questions like who determines the set of alternatives or criteria to be used in comparisons are overlooked. Similarly, we do not consider strategic behavior by the individuals. Our focus is more narrow, but nonetheless important. We deal with various conceptualizations of the notion of "socially best alternative" or "the most defensible collective preference ranking of alternatives". We

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shall look at BC variations from the viewpoint of improving upon BC: are the variations significantly superior to BC in expert group choice settings? As an example of such a setting is the choice of projects to be funded by a funding organization (e.g. EU) on the basis of their evaluation by a group of experts (referees). We are considering ways of utilizing the expert information in richer ways than today is often the case in funding organizations where the experts are asked which applications are acceptable or which are the best. Some of our criteria also pertain to decentralized settings where the experts form subgroups each considering the same set of applications. Some other BC variations have been discussed in an earlier article (Nurmi, 2007).

The relevance of the following discussion extends wider than group choice settings. To wit, all of what is going to be said regarding experts or voters can be translated into another setting involving multiple performance criteria. Substituting criteria for experts or voters we can discuss multiple criterion decision problems using the same techniques. Hence even decision settings where one decision maker is assigned the task of selecting best projects or other alternatives using several criteria of evaluation fall within the range of our discussion. The main assumption to be made is that the evaluation of the alternatives on each criterion allows for ordinal measurement. This is a quite common setting since typically only part of the criteria can be measured by ratio or absolute scales of measurement. Hence, the aggregation problem involving ordinal measurement is frequently encountered.

The setting which we are focusing upon – i.e. the choice by a set of experts – is in fact more in line with what the founding fathers of social choice had in mind than the election of candidates to political offices. Condorcet and Borda were preoccupied with jury decision making, i.e. a setting where one aims at a correct choice assuming that each decision maker has some expertise in the issue to be decided. Also C. L. Dodgson was interested in non-political decision making: awarding scholarships to students in Oxford colleges of the 19th century (Black, 1958). In these settings it is quite natural to assume that the experts, judges, referees or evaluators are able to form a priority ranking over the applicants or alternatives and the task is to aggregate those rankings into collective ones in a logically defensible manner. Since the experts, referees, judges etc. are expected to use their specific expertise in forming their opinion on the decision alternatives at hand, it is very important that the information provided by the experts is aggregated in a way that is non-arbitrary and satisfies plausible criteria of consistency and adequacy.

Among ordinal aggregation methods BC provides a useful benchmark in utilizing fully the information contained in preference or performance rankings. Many other methods – e.g. plurality or antiplurality rules – use only partially the information given in rankings. We will use BC itself as a kind of benchmark method among the positional systems, i.e. among rules that determine the “electability” of an alternative on the basis of its average position in the experts’ preference schedules. The way the social preference ranking is determined on the basis of the individual rankings is very straight-forward in BC: each rank position is assigned a value in a scale so that the difference of values of two consecutive positions equals unity. Prima facie, it is a logically defensible method. BC or slight variations of it are used in Eurovision song contests, making nomination proposals for the offices of university chancellors, bishops etc. In political elections, BC is used in the Pacific island state of Kiribati.

In a way, BC is well-suited as a benchmark system among the positional ones since it provides a neutral and anonymous treatment of all rank positions and individuals. Unless one has a specific reason to put more weight to some positions or individual judges, this seems a reasonable way to proceed. That BC may end up with Condorcet failures – i.e. not rank the Condorcet winner first in the collective preference – may then come as an unpleasant surprise. Hence, one might look for ways of improving the performance of BC by making some modifications in its operating principles. And indeed, more than a hundred years ago Nanson proposed a system that guarantees the choice of an eventual Condorcet winner while otherwise exhibiting the crucial features of BC (Nanson, 1883). It later turned out that the improvement came with a cost: Nanson's rule is non-monotonic, while BC is monotonic. In other words, in Nanson's method an improvement in an alternative's ranking, ceteris paribus, may worsen its position in the collective ranking. This is not possible under BC.

Observations like this made us curious: are there any positional systems "close" to BC that would improve upon its properties while retaining its plausible features? Since BC is based on arithmetic average positions of alternatives, we ask whether BC modification in terms of geometric averages would be an improvement. Similarly, we ask whether replacing averaging by looking at median positions would improve the performance of BC. These are but two possible modifications of BC which are intuitively "close" to BC proper. Together with results on other BC variations, our results, summarized in Table 10, suggest that BC is indeed superior to its modifications among positional systems.

We start by introducing the notation and basic definitions. We then define the modifications of BC to be studied. The criteria of evaluation of the modifications are then discussed.

2. Preliminaries

A *preference aggregation problem* is a triple $A = (N, X, R^N)$, where N is a nonempty finite subset of natural numbers $\mathbb{N} = \{0, 1 \ldots\}$, X is a nonempty finite set, and $R^N = (R_i)_{i \in N}$ is a *profile* of *preferences* R_i on X. A preference R on X is a binary relation satisfying *transitivity* (xRy and yRz implies xRz) and *completeness* (xRy or yRz for all $x, y \in X$). As usual, we denote the membership relation in this context by xRy instead of $(x, y) \in R$. The strict part of a preference R is denoted by P (xPy iff xRy and not yRx), and the indifference is denoted by I (xIy iff xRy and yRx). A preference R is *strict* or *linear*, if xRy and yRx implies $x = y$, for all $x, y \in X$. We denote the set of all preference aggregation problems (or simply problems) by $\mathscr{A}$.

An interpretation of the model is that N is the set of agents, X is the set of states of alternatives, and xR_iy means that agent i prefers at least weakly the alternative x to the alternative y.

An *aggregation rule* is a function f such that $f(A)$ is a preference on X for each aggregation problem $A \in \mathscr{A}$. The interpretation is that $f(A)$ is the social preference representing the tastes of the of the agents in the problem A. There is no shortage of aggregation rules. For example, let $d(A)$ be the least index in N for any aggregation problem A, and define $f(A) = R_{d(A)}$. This is one form of a *dictatorial rule*. The celebrated Arrow's impossibility theorem states that all aggregation rules, satisfying a set of apparently plausible axioms, are dictatorial. Arrow formulated his theorem for problems with a fixed set N of agents, but the result can be extended to the domain of problems with variable agent sets.

Given a problem $A = (N, X, R^N) \in \mathscr{A}$, let $b_i(x) = |\{y \in X \mid xR_iy\}|$, for each $i \in N$ and $x \in X$. Then $b_i(x)$ is the number of alternatives y that in i's opinion are at most as good as x, in a given problem A (the dependence of b_i on A is not explicitly displayed unless absolutely necessary). The *Borda rule* f^B is defined by

$$f^B(A)(x) = \frac{1}{n} \sum_{i \in N} b_i(x) \tag{1}$$

for each $x \in X$, for each $A = (N, X, R^N) \in \mathscr{A}$. We may call $b_i(x)$ the *Borda point* given to alternative x by agent i. Then $f^B(A)(x)$, the *Borda score* of alternative x, is the average of individual Borda points. Sometimes the Borda scores are defined by taking the sum of individual Borda points, and sometimes the Borda point $b_i(x)$ is defined as the number of alternatives that are strictly worse than x. Often the Borda rule is defined only for problems with linear preferences. For all practical purposes these different variants of the Borda rule are the same.

Given a problem $A = (N, X, R^N) \in \mathscr{A}$, let $n(x,y) = |\{i \in N \mid xR_iy\}|$, for each $x \in X$, for each $A = (N, X, R^N) \in \mathscr{A}$. Then $n(x,y)$ is the number of agents in the problem A who prefer x to y at least weakly (the dependence of $n(x,y)$ on A is not explicitly displayed unless absolutely necessary.) It is well known that

$$f^B(A)(x) = \frac{1}{n} \sum_{y \in X} n(x,y) \tag{2}$$

for each $x \in X$, for each $A = (N, X, R^N) \in \mathscr{A}$. This means that the Borda score of the alternative x is actually the average number of individuals who weakly prefer x to y, as y runs through the alternatives in X. A simple way to see that equations (1) and (2) are equivalent is the following. Let $\tilde{R}_i$ be an $|X| \times |X|$ matrix whose rows and columns are indexed by members of X, and whose (x,y)-cell $\tilde{R}_i(x,y)$ is 1 if xR_iy and 0 otherwise. Then $b_i(x) = \sum_y \tilde{R}_i(x,y)$ is the number of 1's in the x'th row. Therefore by (1), $f^B(A)(x) = (1/n) \sum_i \sum_y \tilde{R}_i(x,y)$. This is the same as constructing first the *outranking matrix* $\tilde{R} \equiv \sum_i \tilde{R}_i$, the sum of the $\tilde{R}_i$-matrices, and then taking the average of the cells of the x'th row of the outranking matrix. But this in turn is $f^B(A)(x)$ as calculated in equation (2).

Instead of taking arithmetic averages of the Borda points $b_i(x)$ or the numbers $n(x,y)$ as in (1) and (2), we could take the medians or geometric averages of these numbers. Given an m-dimensional vector y (or an indexed set $\{y_i \mid i \in I, |I| = m\}$), the median of $M(y)$ of y is calculated as follows. Index the coordinates of y by the numbers $1, \ldots, m$ so that $y_i \leq y_j$ if $i \leq j$, $i, j \in \{1, \ldots, m\}$. If m is odd, then $m = 2k+1$

for some $k \in \mathbb{N}$, and $M(y) = y_{k+1}$. If m is even, then $m = 2k$ for some $k \in \mathbb{N}$, and $M(y) = (y_k + y_{k+1})/2$. The geometric average $G(y)$ of the coordinates of y is $G(y) = \sqrt[m]{\prod_i y_i}$, if all numbers y_i are nonnegative, where $\prod_i y_i$ is the product of the numbers y_i.

For all problems $A = (N, X, R^N)$, let $b(x)$ be the vector $(b_i(x))_{i \in N}$ and let $n(x)$ be the vector $(n(x,y))_{y \in X}$. Applying the median, we define the rules f^{Mb} and f^{Mn} by $f^{Mb}(A)(x) = M(b(x))$ and $f^{Mn}(A)(x) = M(n(x))$, for all $x \in X$, for all problems $A = (N, X, R^N)$. Applying the geometric average, we define the rules f^{Gb} and f^{Gn} by $f^{Gb}(A)(x) = G(b(x))$ and $f^{Gn}(A)(x) = G(n(x))$, for all $x \in X$, for all problems $A = (N, X, R^N)$.

Another variant of the Borda count is obtained upon considering again the $|X| \times |X|$ outranking matrix $\tilde{R}$ of the problem A and focusing on the smallest entry on each row. This is obviously the minimum support an alternative receives in all pairwise comparisons. The order of those minima gives us a ranking over all alternatives. More formally, denote the entry on the row x and column y by $\tilde{R}(x,y)$. For each $x \in X$ let

$$\underline{r}_x = \min_y \tilde{R}(x,y) \quad \text{and} \quad Mm(A) = \{x \in X \mid \underline{r}_x \geq \underline{r}_z, \forall z \in X\}.$$

This is the well-known Simpson-Kramer maximin rule (Simpson, 1969; Kramer, 1977).

Other variations can easily be cooked up. Following the intuition that in decision theory goes under the name Hurwicz's rule, we can fix a number $\alpha \in [0,1]$ and define for each $x \in X$ (Milnor, 1954):

$$\bar{r}_x = \max_y \tilde{R}(x,y) \quad \text{to get} \quad h'(x) = \alpha \bar{r}_x + (1-\alpha)\underline{r}_x.$$

Then, the choice set can be obtained as:

$$H(A) = \{x \in X \mid h'(x) \geq h'(x), \forall z \in X\}.$$

In other words, one maximizes the weighted sum of maximum and minimum entries on each row. With $\alpha = 1$, $H(A) = Mm(A)$. Obviously, both $Mm(A)$ and $H(A)$ allow for ranking over all alternatives, so a social preference can be formed. Both utilize essentially less information about voter preferences than the Borda count and its geometric average variation.

A choice method devised by Litvak (1982) is very much in the spirit of the Borda count. Consider two individuals and their rankings over X. The individuals disagree about the priority of the alternatives to the extent their rankings differ. One way of measuring this disagreement is to sum up the Borda point differences, i.e.

$$\text{dis}(R_1, R_2) = \sum_{x \in X} |b_1(x) - b_2(x)|$$

The values of the *dis* measure range from 0 to $\sum_{i=0}^{k} [(k-1) - (2i)]$, where k is the number of alternatives. Litvak's method looks for a consensus ranking over alternatives that is closest to the expressed opinions (ranking) of individuals in the sense of the *dis* measure. To wit, given a preference profile R^N of n voters, define the distance of a fixed ranking R and R^N as follows:

$$\mathrm{dis}(R^N, R) = \sum_{j \in N} \sum_{x \in X} \mid b_j(x) - b_I(x) \mid .$$

where $b_I(x_i)$ denotes the Borda points assigned to x by the ranking R.

Let $\mathscr{R}$ be the set of all rankings over k alternatives. Given R^N, Litvak's method results in $R \in \mathscr{R}$ where

$$L(A) = \{R \in \mathscr{R} \mid \mathrm{dis}(R, R^N) \leq \mathrm{dis}(R', R^N), \forall R' \in \mathscr{R}\}.$$

Variations of Litvak's method can easily be envisioned. For example, the city-block metric could be replaced by the Euclidean one. Or, one could focus only on the first rank of the consensus ranking and sum up, for each alternative, the difference between the alternative's position and the first rank. This variation, however, leads us back to the Borda count (Nitzan 1981). This shows not only that with varying consensus profiles and distance measures one is able to construct different methods, but also that choice rules can be expressed in several ways.

Further variations are based on certain entries in the outranking matrix, such as two smallest and two largest ones, two entries closest the the mean one etc. Similarly, the weights assigned to various entries can be varied to end up with new variations. Instead of defining these largely ad hoc rules, we shall focus our analysis on the rules explicitly defined above. Our basic interest is in finding out whether any of these can be considered an improvement over the original version, viz. the Borda count.

3. Properties of Borda variations

3.1 Eliminating Condorcet losers

Arguably the primary motivation for BC was the fact that it never elects the eventual Condorcet loser in the reported profile. In other words, whenever the profile expressed by the voters contains an alternative that would be defeated by all other alternatives in pairwise majority comparisons, this alternative is not the BC winner. Had Borda been introduced to the median Borda count f^{Mb}, he would, therefore, have been largely unimpressed since it turns out that this system may end up with a Condorcet loser. The following 7-voter profile is illustrated in Table 1. Here B is the Condorcet loser and is at the same time the f^{Mb}-winner.

Table 1. Median BC f^{Mb} may elect a Condorcet loser

1 voter	ABCDE
1 voter	CBADE
1 voter	DBEAC
1 voter	EBADC
1 voter	ADCEB
1 voter	ACDEB
1 voter	CDEAB

Let there be k alternatives and n voters. When all voter preferences are strict and the number of voters odd, we can form a $k \times k$ matrix $T = [t_{xy}]$, $x, y \in X$, of 0's and 1's so that a 1 in row x and column y means that alternative i is preferred to alternative j by more than 50% of voters. Otherwise, the entry is 0. If n is odd and all voter preference strict, $t_{xy} = 1$ implies that $t_{yx} = 0$ when $x \neq y$. Thus, the number of 1's in T is $w = k \times (k-1)/2$.

We now show that the median Borda system f^{Mn} based on outranking numbers does not result in an eventual Condorcet loser being elected.

Proposition 1. *Let n be odd and all preferences strict. If T contains a row x so that $\sum_y t_{xy} \geq k/2$, then the Condorcet loser cannot win if f^{Mn} is used.*

Proof. The proof is immediate upon observing that if the number of 1's is at least $k/2$, the median entry in row x of the corresponding outranking matrix is strictly larger than $n/2$, while the median entry on the row corresponding to the eventual Condorcet loser is strictly less than $n/2$. Hence, the Condorcet loser is not chosen when the f^{Mn} system is used. □

Proposition 1 establishes a sufficient condition for the Condorcet loser not being chosen. The next proposition shows that whenever there is a Condorcet loser, the condition is satisfied.

Proposition 2. *Assume that n is odd and all preferences are strict. If there is a Condorcet loser in the problem $A = (N, X, R^N)$, then there is a row x such that $\sum_y t_{xy} \geq k/2$.*

Proof. Suppose to the contrary that for all x, $\sum_y t_{xy} < k/2$, and so $\sum_y t_{xy} \leq (k-1)/2$. Therefore

$$\sum_x \sum_y t_{xy} \leq k \times (k-1)/2 = w.$$

But this must be satisfied as an equality by the definition of w. So $\sum_y t_{xy} \geq (k-1)/2$ and there is no Condorcet loser, a contradiction. □

The maximin method, in contrast, may end up with a Condorcet loser, as shown in the example of Table 2. Here D, the Condorcet loser, gets the minimum support of 14 which exceeds that of all other alternatives. Litvak's method can also lead to the choice of a Condorcet loser (Nurmi, 2004, p. 9).

Consider now the maximax method, i.e. a method that results in the choice of the alternative with the largest maximal element in its row in the outranking matrix. It is apparent that whenever there is a Condorcet loser in the observed profile, it cannot

Table 2. Maximin method may elect a Condorcet loser

10 voters	DABC
8 voters	BCAD
7 voters	CABD
4 voters	DCAB

Table 3. Both f^{Gb} and f^{Gn} may elect a Condorcet loser

1 voter	ABCDEFGHIJ
1 voter	JABCDFEGHI
1 voter	IJABCFDEGH
1 voter	HIJABFCDEG
1 voter	GHIJAFBCDE
1 voter	EGHIJFABCD
1 voter	DEGHIFJABC
1 voter	CDEGHFIJAB
1 voter	BCDEGFHIJA

be elected under the maximax method. This follows from the definition of Condorcet loser. Since it is defeated by a majority by every other alternative, this means that every other alternative has a larger element in its row than any element in the Condorcet loser's row. Consequently, the latter cannot win.

Now, since the maximin may lead to the choice of a Condorcet loser, while the maximax method never ends up with one, it follows there are Hurwicz type methods, i.e. ones based on weighted average of the minimum and maximum entries on each row of the outranking matrix, that necessarily exclude the Condorcet loser and also methods of the same type that may choose the Condorcet loser.

Consider next the method of forming the social preference by comparing geometric averages of either the rows of the outranking matrix (f^{Gn}) or the Borda points given to alternatives by the voters (f^{Gb}). It turns out that the Condorcet loser may be selected as the unique socially best alternative by both methods. Take a look at Table 3, where the strict preferences of nine voters over ten alternatives are depicted.

The alternative F is a Condorcet loser, every other alternative beats it by votes 5 - 4. So the row of the 10×10 outranking matrix corresponding to alternative F consists of nine 4's and one 9 (F is at least as good as itself in the eyes of all voters). The product of these numbers is therefore $4^9 \times 9$. The other rows of this matrix consist of the whole numbers from 1 to 9, the number 5 appearing twice. The product of these numbers is $9! \times 5$. Since $4^9 \times 9 > 9! \times 5$, the Condorcet loser is on the top of the social preferences when geometric averages of the rows of the outranking matrix are compared, i.e. when the f^{Gn} rule is used.

Now $b_i(F) = 5$ for all voters i, and the product of these numbers is 5^9. For all other alternatives x the Borda points $b_i(x)$ are all natural numbers between 1 and 10 except 5. So the product of Borda points for all alternatives $x \neq F$ is $10!/5$. Since $5^9 > 10!/5$, the Condorcet loser is on the top of the social preferences when geometric averages of the Borda points of alternatives are compared, i.e. when the f^{Gb} rule is used.

3.2 Consistency-related properties

The properties we shall focus upon are *separability* (Smith, 1973), *consistency* (Young, 1974), *faithfulness* (Young, 1975) and *positive involvement* (Saari, 1995, p. 216).

Separability requires that if two voter groups V_1 and V_2 both prefer alternative a to alternative b, then so does the combined group $V = V_1 \cup V_2$. Moreover, if one of the group preferences is strict, this is also the case for the preference of V.

Consistency states that if two disjoint groups of individuals, V_1 and V_2 when choosing from the same set of alternatives X, choose the same alternatives X' (and possibly some others as well), then the group $V = V_1 \cup V_2$ should choose X'. Young points out that consistency is a version of Pareto optimality for subgroups since X' only is preferred to all other alternatives by the subgroups V_1 and V_2 considered as individuals.

Faithfulness is an intuitively compelling property (Young, 1974). It states that if the voting body consists of only one individual, then the social preference ranking is identical with the individual's preference ranking. Now, if a system is both faithful and consistent, then it also satisfies unanimity or Pareto condition, i.e. in profiles where all individuals agree on the first ranked alternative, this alternative is chosen.

A procedure is positively involved if, whenever a is chosen by V, it is also chosen if a group of voters, with identical preferences so that a first ranked, joins V.

The above properties are pretty close to each other, but by no means equivalent. Saari (1995, p. 218) shows that any scoring rule that is faithful and consistent is also positively involved, but not all faithful and positively involved scoring rules are consistent.

It is known that BC is faithful and consistent (Young, 1974). Hence it is also positively involved and satisfies the Pareto condition. The median BC f^{Mb}, in contrast, is not consistent. This is illustrated by Table 4, where the profile above the middle horizontal line denotes the profile of V_1 and that below the line depicts V_2's profile. The choice sets are $\{A,B\}$ and $\{A\}$, respectively. Their intersection is obviously $\{A\}$, but f^{Mb} specifies $\{A,B\}$ as the choice set in $V_1 \cup V_2$.

The other median Borda variation, f^{Mn} is not consistent either. This is shown by Table 5. The choice set from the profile above the middle line is $\{A\}$ and $\{A,C\}$ from the profile below it, while the entire profile of 6 voters ends up with $\{A,C\}$.

Young's (1975) theorem on social choice scoring functions states that all anonymous, neutral and consistent procedures fail on the Condorcet winning criterion. Hence, if the maximin method – which is anonymous and neutral – were consistent, it would have to fail on the Condorcet winner criterion. Maximin is, however, not a scoring rule. So, even though it fails on the Condorcet winner criterion, we are not entitled to

Table 4. f^{Mb} is not consistent

1 voter	ABCD
1 voter	ABDC
1 voter	BACD
1 voter	BCDA
1 voter	ABCD
1 voter	ABDC
1 voter	DABC
1 voter	CDBA
1 voter	CDAB

Table 5. f^{Mn} is not consistent

1 voter	ACBD
2 voters	BACD
1 voter	ACBD
2 voters	CABD

Table 6. Maximin is not consistent

4 voters	ABC
3 voters	BCA
3 voters	CAB
5 voters	ACB
4 voters	CBA

the conclusion that it is inconsistent. Table 6, however, shows by way of an example that it is inconsistent. The maximin choice set in the upper and lower parts consists of A, while the choice from the entire 19 voter profile is C.

Maximax rule is not consistent either. This is shown in Table 7. The maximax choice sets are $\{C\}$ and $\{A,C\}$, but the choice from the combined profile is $\{A\}$.

Both geometric BC variations, f^{Gb} and f^{Gn}, in turn, are consistent. This results from the observation that whenever the product of the Borda scores or outranking numbers are largest in V_1 and in V_2, the product of those products must also be larger than the corresponding product for any other alternative.

Also Litvak's method turns out to be inconsistent as shown by the example of Table 8. Denote the part above the middle line as V_1 and that below the middle line as V_2. Then the choice sets of V_1, V_2 and V are $\{B\}$, $\{A,B\}$ and $\{A\}$, while consistency would dictate $\{B\}$ as the choice set from V.

Turning to faithfulness we observe that BC satisfies this property (Young, 1974). So does f^{Mb} since the individual's highest ranked alternative has trivially the highest median Borda score, the second-ranked the next highest and so on. In contrast, f^{Mn} is not faithful. This can be seen by considering a 3-alternative case and ignoring the diagonal entries of the outranking matrix. The f^{Mn} choice set consists of the first and second ranked alternatives, a contradiction with faithfulness.

One of the geometric Borda variations is not faithful, viz. f^{Gn}. This is easily seen

Table 7. Maximax is not consistent

3 voters	ABC
3 voters	BCA
3 voters	CAB
1 voter	CBA
2 voters	ACB

Table 8. Litvak's method is not consistent

5 voters	ABC
4 voters	ACB
2 voters	BAC
5 voters	BCA
3 voters	CBA
2 voters	ABC
2 voters	BAC

since all rows in the outranking matrix contain at least one zero with the sole exception of the row that represents the first-ranked alternative. Hence, the f^{Gn} ranking is a dichotomous one: first-ranked alternative first with the rest forming a tie. The other geometric variation f^{Gb}, on the other hand, is faithful since the collective ranking coincides with that of the only individual forming the collectivity.

The maximin method fails on faithfulness as well since all rows except that corresponding to the first-ranked alternative contain a zero, whereby all these alternatives tie for second place even in cases where the individual has a ranking without any ties. Similarly, the maximax is not faithful either.

Litvak's method, in its turn, is faithful as the individual's ranking is at the shortest distance from itself. Thus, this ranking is also the collective one.

Positive involvement is satisfied by BC since it is a scoring rule and satisfies faithfulness and consistency. The geometric Borda rule f^{Gb} – the rule that maximizes the geometric average of the product of Borda points given to each alternative – is also positively involved since adding an individual with preference ranking that coincides with the collective one entails multiplying the score of the winning alternative with the largest individual Borda score, the second-ranked alternative with the second largest Borda point etc. Hence the new score of the winning alternative is larger than that of any other alternative.

The other geometric version f^{Gn} is also positively involved. For suppose that a is the first ranked alternative in a profile of V_1 and then a group V_2 of individuals with identical preferences and a first ranked joins the profile. In the outranking matrix of V_1 alternative a's column consists of zeros only. This is also the case in V_2's outranking matrix. This means that the only row with non-diagonal entries all greater than zero in $V_1 \cup V_2$ is a's row. The product of its entries is then the only one that differs from zero. Hence, a is elected by $V_1 \cup V_2$.

The median Borda variation, f^{Mb}, is not positively involved. This is shown in Table 9. The upper part represents the V_1 profile where C wins. Adding now the lower part profile where C is first ranked yields A the winner.

The median Borda variation f^{Mn}, in contrast, is positively involved. This follows from the fact that the winning alternative a's row elements in the outranking matrix will be added by the number of individuals in V_2. This means that the median entry on a's row in the outranking matrix of $V_1 \cup V_2$ is added by the number of individuals in V_2. In other rows the entries will be added by $|V_2|$ or not at all. In these rows the median

Table 9. f^{Mb} is not positively involved

2 voters	AFCDBEG
1 voter	EDCABFG
2 voters	BGCDEAF
2 voters	CABDEFG

entries will be no larger than the corresponding median values in V_1 plus $|V_1|$. Hence the choice of f^{Mn} remains the same after a group of identically minded individuals with a their first ranked alternative joins V_1. Thus, f^{Mn} is positively involved.

Litvak's method is positively involved. The proof can sketched as the following *reductio ad absurdum* argument. Assume that a is the Litvak winner (i.e. first ranked) in V_1. Assume, moreover, that V_2 consists of individuals with identical preference rankings so that a is first in this consensus ranking. Finally, assume that the ranking R where $x \neq a$ is the one that is at the minimal (Litvak) distance from the rankings in V_1 and V_2. The trick is to show that R can be improved upon, i.e. that it is not in fact the ranking that is closest to the V_1 and V_2 rankings. This is seen by switching a and x in R to obtain R' and observing that R' is closer to the rankings of V_1 and V_2 than R. Hence, R is not at minimal distance. Hence, the claim that Litvak's method is positively involved follows.

Also the maximin method is positively involved. If a wins in V_1 and is joined by V_2 of individuals with identical preferences so that a is first ranked, each entry of a's row in the outranking matrix of $V_1 \cup V_2$ is added by $|V_1 \cup V_2|$, while only some entries of other rows are similarly added. Hence, the minimal entry on a's row remains the largest.

Similar argument shows that the maximax method is also positively involved.

4. Conclusion

Table 10 summarizes the results of the preceding. The main overall conclusion is that BC beats the other systems discussed in this paper hands down in terms of the criteria we have dealt with. A reader with a more "binary" or "Condorcetian" persuasion might wonder why we haven't included the Condorcet winning criterion into the picture. This well-known condition states that if there is a Condorcet winner in the profile under investigation, this alternative should be chosen. Saari's (e.g. Saari, 1995; Saari, 2006) works have cast a shadow over this criterion and, hence, we have not used it here. Another reason is that the Condorcet winning criterion often plays a crucial role in various incompatibility results – such as incompatibility of Condorcet criterion and invulnerability to the no-show paradox (Moulin, 1988) or of Condorcet criterion and nonmanipulability (Gärdenfors, 1976). Therefore, if the Condorcet criterion is in doubt, much of the practical importance of these results is swept away.

Of course, it is not necessary to conduct project evaluations with BC or any of its modifications. We re-emphasize, however, the intuitively appealing features of BC in these contexts: all alternatives are handled neutrally, all voters anonymously and

Table 10. Summary assessment of methods

Criterion method	C-loser exclusion	Consistency	Faithfulness	Positive involvement
BC	yes	yes	yes	yes
f^{Mb}	no	no	yes	no
f^{Mn}	yes	no	no	yes
f^{Gb}	no	yes	yes	yes
f^{Gn}	no	yes	no	yes
maximin	no	no	no	yes
maximax	yes	no	no	yes
Litvak	no	no	yes	yes

all positions have an equal value attached to them. Moreover, the difference between values of two consecutive positions is constant. The well-known drawback of BC that it is vulnerable to introduction of "phantom" options (irrelevant alternatives) can in the project evaluation contexts be largely ignored since the contestant project set is usually fixed before the expert evaluation begins.

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