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CENTRALIZATION TRADE-OFF WITH NON-UNIFORM TAXATION

Abstract

When local public goods are provided by a centralized authority, spillovers are internalized, but heterogeneity in preferences may be suppressed. Besley and Coate (2003) recently examined this classic trade-off for a uniform tax regime with strategic delegation. Here, we extend their approach by allowing for a non-uniform tax regime. We find that centralization with non-uniform taxation unambiguously increases welfare in comparison to uniform-tax centralization. With non-cooperative legislators coming from symmetric districts, our centralization dominates decentralization for any degree of spillovers. In other cases, non-uniform taxation at least improves the odds of centralization, if measured by a utilitarian yardstick.

Keywords: decentralization, taxation, local public goods

JEL classification: H40, H70, H72, P51

1. Introduction

Centralization involves a host of trade-offs. A centralized government exploits returns to scale in public goods provision, but faces pronounced information asymmetry. Spillovers can be efficiently resolved by centralization, but centralized taxation may be more distortive. Also, centralization may induce higher total investment into wasteful rent-seeking. The case for centralization of local public goods is predominantly driven by cross-border spillovers, and is strengthened in the presence of economies of scale in public good production (or in tax administration, cf. (Redoano and Scharff, 2004)).

However, centralization is not costless. As Oates (1972) argues, welfare loss arises whenever jurisdictions vary in preferences, and the amount of a public good is bound to be equal across jurisdictions. The loss magnifies with increasing price elasticity of public good demand. Secondly, the issue of asymmetric information is of extreme relevance here. For a central authority, reliable local information is costlier to obtain. Voters observe lower yardstick competition at the central level and, hence, have fewer opportunities to assess public sector performance.

Seabright (1996) builds a model capturing another important problem of centralization, namely lower accountability. Centralization may decrease the level of political

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competition, providing an additional source of political rents. Furthermore, the proponents of the FOCJ concept (Functionally Overlapping Competing Jurisdiction), mainly Frey (1996), observe slower policy innovation in a centralized system. On the other hand, Kotsogiannis and Schwager (2001) construct a model with opposite results.

When interest groups in jurisdictions have common interests and lobbying features economies of scale, centralization may also boost rent-seeking expenditures and consequently distort market allocation (Bordignon *et al.*, 2003; Bardhan, Mookherjee, 2000).

To tackle the benefits and costs of centralization, we restrict attention to the classic trade-off between spillovers and heterogeneity. We put aside other, albeit important, issues of locational choice, accountability, and incomplete information. Our approach draws from the seminal setup of Besley and Coate (2003), but we extend their approach by introducing a non-uniform tax system. In other words, we study how centralization performs when the benefits (i.e. amounts of local public goods) as well as the costs are non-uniform across districts.

Besley and Coate (2003) depart from the existing literature in emphasizing the *political processes* of decision-making. If governments under centralized systems were allowed to allocate different levels of local public goods to different districts, they could respect the preferences of citizens in each district while optimally accounting for cross-border spillovers. This would make the centralized system preferable. If there is a case for a decentralized system, then it must follow from political economy considerations.

Centralization has been typically modeled as a system in which public spending is financed by general taxation and all jurisdictions receive a uniform level of local public goods. In a decentralized system, local public goods are financed by local taxation and each district chooses its own preferred level. This approach has been adopted by Oates (1972), who argued that the drawbacks of centralized and decentralized systems are uniformity in provision and absence of reflecting the benefits going to other regions, respectively. This logic relies crucially on the assumption that centralization provides uniform levels of public goods. Besley and Coate (2003) relax this assumption and then study various forms of centralized decision-making.

We also relax the assumption of policy uniformity (i.e. uniform level of public goods) stipulated in Oates (1972) and Musgrave (1959). To the best of our knowledge, this assumption was only motivated by the implications of asymmetric information incentives between central and local authorities.[1]

The main innovation of our approach is to let district-specific head tax depend on the amount of public goods provided in each region. The reason is that for district-uniform head tax, a marginal increase in public spending in one district increases average tax across all districts equally. In our tax system, the increase is not equal, but higher for the beneficiary district. This brings the private marginal cost of public goods provision closer to the private marginal benefit and improves the odds for centralization compared to decentralization.

For a centralized legislature, we postulate either non-cooperative or cooperative policy makers. We observe that cooperativeness as such cannot resolve conflicts in preferences among the regions, since voters can insure against policy cooperativeness by strategic delegation. Strategic delegation has its origins in industrial economics (Fershtman, Judd, 1987) and monetary policy (Rogoff, 1985), resulting in applications

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[1] Cheikbossian (2000) argues that in a game played between a central policy-maker and decentralized authorities, the latter have an incentive to report excessive expenditure needs and low tax-paying capacity. This misinformation induces policy uniformity.

to public economics (Persson, Tabellini, 1990). Like in Cheikbossian (2000), in our framework a non-cooperative voter delegates a policy maker with a different concern for local public goods than is his or her own.

The remainder of the paper is organized as follows. Section 2 outlines the framework for our analysis. Section 3 provides a brief review of the standard analysis. Section 4 presents a political economy analysis with two forms of taxation beginning with a centralized system, assuming minimum winning coalitions. Section 5 continues in this direction, considering a more cooperative legislature. Finally, Section 6 offers concluding remarks.

2. The Model

There are two geographically distinct regions or districts indexed by $i \in \{1,2\}$, each populated by a continuum of citizens with a mass of unity. The citizens are immobile between the regions. The economy contains three goods: a single private good, x, and two local public goods, g_1 and g_2, each one associated with a particular district. Each citizen is endowed with some of the private good and throughout we will assume that the endowments are high enough for each citizen to meet their required tax obligations. To produce one unit of either of the public goods requires (constant) p units of the private good.

Each citizen in district i is characterized by a public goods preference parameter λ, to be interpreted as the interest in public goods of both districts. The preferences of a type λ citizen in district i are

$$x + \lambda [(1 - \kappa) \ln g_i + \kappa \ln g_{-i}],$$

where parameter $\kappa \in [0, 1/2]$ indexes the degree of spillovers.[2] When $\kappa = 0$, citizens consume only the public good in their own district, while for $\kappa = 1/2$ they equally consume public goods in both districts. Postulating uniform κ (across regions) means that citizens from both districts consume both public goods in the same proportion.[3]

The range of preference types is $\lambda \in \langle 0, \lambda^{max} \rangle$ in each district. The respective median type in district i is denoted by m_i. We assume, without loss of generality, that the median citizen in district 1 is at least as pro-public spending as his counterpart in district 2, i.e. $m_1 \geq m_2$. We also assume that $2m_1 < \lambda^{max}$. The latter condition will be needed in Section 5 to obtain interior solutions.

Under a *decentralized system,* the level of public goods in each district is chosen by the government of that district and public goods are financed by a uniform head tax on local residents. Thus, if district i chooses a public good level g_i, each citizen in this district pays a tax of pg_i. Under a *centralized system,* the levels of public goods are

[2] For more universal specifications of public goods preferences, see (Besley, Coate, 1999, 2000)).

[3] It could be that citizens from district 1 cared equally about the public goods in both districts, whereas citizens from district 2 cared only about their own public good. That is to say, proportional public goods preferences κ could differ among regions. Moreover, differences among citizens from the same district could occur, i.e. a citizen could, for instance, derive benefit from both public goods, but his neighbor from the same district could be interested only in the public good provided by his own district. We will, however, assume these spillovers to be of a purely technical nature. Moreover, we will assume them to be the same for citizens from both districts as well as for the citizens in a given region, so as to be able to capture the fact that the resolution between centralization and decentralization depends on the (symmetric) degree of spillovers and the extent of heterogeneity in preferences for public goods.

determined by a government that represents both regions. Spending is being financed by two possible tax systems, whose outcomes will be compared. The first one is a *uniform* head tax on all citizens; with public good levels g_1, g_2, this tax is $p(g_1 + g_2)/2$. The second one is a head tax which is *non-uniform* across districts, but uniform for all citizens within a given region. A citizen of each region pays a head tax proportional to his consumption of both public goods; thus, public goods levels g_1, g_2 and degree of spillovers κ result in a head tax of $pg_i(1 - \kappa) + pg_{-i}\kappa$ in district i.

Our social welfare criterion for comparing the performance of centralized and decentralized provision of local public goods will be the aggregate public goods surplus. With public goods levels (g_1, g_2), it is defined as

$$S(g_1, g_2) = [m_1(1 - \kappa) + m_2\kappa] \ln g_1 + [m_2(1 - \kappa) + m_1\kappa] \ln g_2 - p(g_1 + g_2).$$

The surplus is a median citizens' sum of the marginal benefits from their consumption of public goods or, more precisely, a sum of the differences between median citizens' utilities under the system with public goods and the system without public goods:

$$\Delta U_{m_1} + \Delta U_{m_2} = x_1 - pg_1 + m_1[(1 - \kappa) \ln g_1 + \kappa \ln g_2] - x_1 + \\ + x_2 - pg_2 + m_2[(1 - \kappa) \ln g_2 + \kappa \ln g_1] - x_2 = S(g_1, g_2)$$

The surplus-maximizing public goods levels[4] are as follows:

$$(g_1, g_2) = \left(\frac{m_1(1 - \kappa) + m_2\kappa}{p}, \frac{m_2(1 - \kappa) + m_1\kappa}{p} \right)$$

This result reveals that the surplus-maximizing public goods levels take account of the benefits received by citizens from both districts.

3. Classic Setup

The model outlined above allows a simple exposition of the traditional analysis according to Oates (1972), who influenced many public finance economists' views on the relative merits of centralization and decentralization. He supposed that, in a decentralized system, each district's government independently chooses the policy which maximizes the public goods surplus in the region (which is ΔU_{m_i}, $i \in \{1, 2\}$). A pair of expenditure levels (g_1^d, g_2^d) will form a Nash equilibrium, which requires that:

$$g_i^d = \arg \max_{g_i} \{m_i[(1 - \kappa) \ln g_i + \kappa \ln g_{-i}^d] - pg_i\}, \quad i \in \{1, 2\}.$$

Taking first-order conditions yields:

$$(g_1^d, g_2^d) = \left(\frac{m_1(1 - \kappa)}{p}, \frac{m_2(1 - \kappa)}{p} \right)$$

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[4] The vector is an interior solution of a simple maximization of function $S(g_1, g_2)$. We obtain it by taking first-order conditions, i.e. by differentiating this function w.r.t. g_1 and g_2 and setting it equal to zero. It is straightforward to verify that the second-order conditions are satisfied and we leave this proof to the reader. This remark applies to all of the subsequent maximization problems.

Each region's government thus only takes into account the benefits received by its constituency and local public goods are surplus-maximizing only when there are no spillovers, regardless of heterogeneity in tastes. When spillovers occur, public goods production results in under-provision in both districts and this under-provision is increasing in the extent of spillovers.

Under a centralized system, Oates assumed that the government would be restricted to provide a uniform level of public goods, denoted g^c. He further assumed that expenditures would be financed by a uniform head tax, which is, in the case of uniform provision of public goods, identical to our proposed non-uniform head tax that takes into account the proportional consumption of both goods by citizens from each region.[5] This common level of public goods satisfies
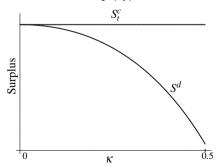
$$g^c = \arg \max_g \{[m_1 + m_2] \ln g - 2pg\} = \frac{m_1 + m_2}{2p}.$$

The uniform level of public goods is independent of the level of spillovers and results in the surplus-maximizing level only in the case of identical districts.[6] However, when $m_1 > m_2$, centralization over-provides public goods to district 2 and under-provides them to district 1 except when spillovers are maximal, i.e. $\kappa = 1/2$. In this situation, citizens consume public goods in both districts equally, which leads to uniform provision of public goods in both regions.

3.1 Comparative Statics

When regions are homogeneous, centralization produces surplus-maximizing public goods levels and dominates decentralization whenever spillovers are present. Centralization has the conventional advantage of internalizing spillovers, as *Figure 1* illustrates.

FIGURE 1 Aggregate Public Goods Surpluses under Decentralization (S^d) and Centralization in the Classic Setup (S_t^c): Identical Districts
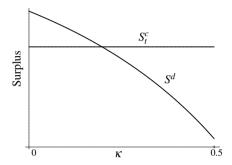

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[5] This stems from the simple fact that, with uniform provision and identical prices of local public goods, Oates' head tax $p(g + g)/2 = pg$ equals $pg(1 - \kappa) + pg\kappa = pg$.

[6] Throughout the text, the phrases *identical* (*non-identical*) and *homogeneous* (*heterogeneous*) *districts* will indicate that the median citizens from each region *have* (*do not have*) the same public goods preferences.

In the case of heterogeneous districts, decentralization yields surplus-maximizing public goods levels and dominates centralization when there are no spillovers. In contrast, when the spillovers are maximal, centralization produces surplus-maximizing public goods levels and dominates decentralization. When the spillovers are in between these two polar cases, there exists a critical level of spillovers above which centralization dominates and under which decentralization is preferred; see *Figure 2*.

FIGURE 2   Aggregate Public Goods Surpluses under Decentralization (S^d) and Centralization in the Classic Setup (S_t^c): Non-Identical Districts

Proposition 1. *Suppose that the assumptions of the standard analysis are satisfied.*

(i) *If the regions are homogeneous and spillovers are present $(\kappa > 0)$, a centralized system produces a higher level of surplus than does decentralization. In the absence of spillovers $(\kappa = 0)$, the two systems generate the same level of surplus.*

(ii) *If the districts are heterogeneous, there is a critical value of κ, greater than zero but less than $1/2$, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level.*[7]

According to Oates, without spillovers, a decentralized system is preferred. With spillovers and homogeneous districts, a centralized system is superior. With spillovers and heterogeneous regions, it is necessary to compare the extent of the two effects.

It is often suggested that heterogeneity favors the case for decentralization. In our model, this does not follow immediately, since we cannot conjecture that the critical level of spillovers increases in heterogeneity.[8] In any case, modeling the trade-off between centralization and decentralization in the classic setup relies on the assumption of uniform expenditures under centralization, which is a too restrictive assumption; henceforth we expand the model to include the possibility of non-uniform provision.

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[7] The proof of this, as well as the other results, may be found in the appendix.

[8] This may be analyzed by letting $S^d(\kappa, \alpha)$ and $S_t^c(\kappa, \alpha)$ denote surpluses under decentralization and centralization, respectively, when $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$, where $\alpha \in \langle 1/2, 1 \rangle$ measures the degree of heterogeneity between the two regions. Districts are identical when $\alpha = 1/2$ and become more heterogeneous when α increases. Then, $S_t^c(\kappa, \alpha) = \omega \ln(\omega/2p) - \omega$, which is independent of both κ and α. Therefore we can write $S_t^c(\kappa, \alpha) = S_t^c$. The critical value of κ, denoted $\kappa^*(\alpha)$, is uniquely defined by the equation $S^d(\kappa^*, \alpha) = S_t^c$. To show that κ^* is an increasing function of α, it is necessary to show that for all α,

4. Political Economy with Two Forms of Taxation

4.1 Decentralization

In a decentralized system, we assume that each region elects a single representative from that region to choose policy. Our model is based on the citizen-candidate approach to political decision-making, which has two stages. First, elections determine which citizen from each district is selected to constitute the decision-making government in that district (election stage). Second, policies are chosen simultaneously by the elected representatives in each district (policy-selection stage).

Using backward induction,[9] we proceed as follows. First, we find what the elected representatives select (stage 2 or the policy-selection stage) and then we discuss whom citizens, considering outcomes which are subsequently selected by the representatives, will appoint to an office (stage 1 or the election stage). Beginning with stage 2, let the types of the representatives in district 1 and 2 be λ_1 and λ_2, respectively.[10] Then the policy outcome $(g_1(\lambda_1), g_2(\lambda_2))$ satisfies

$$g_i(\lambda_i) = \arg\max_{g_i}\{\lambda_i[(1-\kappa)\ln g_i + \kappa\ln g_{-i}(\lambda_{-i})] - pg_i\}, \quad i \in \{1,2\}.$$

Solving this with first-order conditions yields

$$(g_1(\lambda_1), g_2(\lambda_2)) = \left(\frac{\lambda_1(1-\kappa)}{p}, \frac{\lambda_2(1-\kappa)}{p}\right).$$

the level of each district's public goods spending is higher the stronger is the public good preference of its representative and lower the higher is the level of spillovers.

Now let us move to stage 1. With the representatives λ_1 and λ_2 in region 1 and 2, respectively, a citizen of type λ in district i will enjoy a public goods surplus

$$\Delta U_{\lambda,i} = \lambda\left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)}{p}\right] - \lambda_i(1-\kappa).$$

These preferences over types determine citizens' voting decisions. A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred under decentralization if, in each district i, a majority of citizens prefer the type of their representative to any other type $\lambda \in \langle 0, \lambda^{max}\rangle$, given the type of the other district's representative λ_{-i}^*.

$\partial S^d(\kappa^*, \alpha)/\partial\alpha > 0$. Differentiating, we obtain

$$\frac{\partial S^d}{\partial\alpha}(\kappa,\alpha) = \omega(1-2\kappa)\ln\frac{\alpha}{1-\alpha} + \omega\kappa\frac{1-2\alpha}{\alpha(1-\alpha)}.$$

The first term is positive, while the second one is negative. As spillovers increase, the first term goes to zero. Thus, it is possible that $\partial S^d(\kappa,\alpha)/\partial\alpha < 0$. (In our specification of public goods preferences, the surplus under decentralization is always decreasing in heterogeneity for all $\kappa > 1/4$. This finding makes it possible that the critical level of spillovers is decreasing in heterogeneity, i.e. The case for centralization could be strengthened as the regions become more diverse.)

[9] Backward induction is an iterative process for solving finite extensive-form games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next-to-last moving player is determined taking the last player's action as given. The process continues in this way backwards in time until all players' actions have been determined. Effectively, one determines the Nash equilibrium of each subgame of the original game.

[10] We assume that candidates have no opportunity costs, i.e. any citizen can agree to be a candidate; and that representatives can only decide on the provision of public goods, i.e. there are no other perquisites of the office.

We assume that the elected representatives in the two regions will be of these majority preferred types. Further we assume that each citizen votes sincerely (according to his public goods preferences), does not abstain, and has perfect information.

Citizens' preferences over types are single-peaked,[11] implying that a pair of representative types is majority preferred under decentralization if and only if it is a median pair; i.e. $(\lambda_1^*, \lambda_2^*) = (m_1, m_2)$. This yields:

Lemma 1. *Suppose that the assumptions of the political economy analysis are satisfied. Then the policy outcome under decentralization is*

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)}{p}, \frac{m_2(1-\kappa)}{p} \right).$$

These levels of local public goods respect the preferences of the median citizen within a region, which agrees with the standard local public finance analysis.

4.2 Centralization with Two Forms of Taxation

The policy determination process under centralization also has two steps: an election stage; and a policy selection stage. In the elections, one citizen from each district is chosen to serve in a common legislature. In the policy selection stage, the legislature determines public goods provision in each region. Our first method of capturing the decision-making process in the legislature will be the minimum winning coalition view. Under this view, a coalition of just above 50 % of the representatives forms to share the benefits of public spending among their districts. Regions whose representatives are outside the coalition are only allocated spending to the extent that this benefits coalition members. The logic is that, in a majority rule legislature, if there were any more than just above 50 % of the representatives in the coalition supporting the spending bill, the majority of coalition members would benefit from expelling the surplus members and further concentrating spending on their own regions. Because there are many possible minimum winning coalitions, this view suggests that there will be uncertainty concerning the identity of the coalition that forms to determine expenditures.

In our model, we may capture this uncertainty by assuming that each representative can be thought of as a minimum winning coalition with equal probability. Thus, again using backward induction, if the representatives are of types λ_1 and λ_2, the policy outcome will be $g_1^1(\lambda_1), g_2^1(\lambda_1)$ with probability 1/2 and $g_1^2(\lambda_2), g_2^2(\lambda_2)$ with probability 1/2, where $g_1^i(\lambda_i), g_2^i(\lambda_i)$ is the optimal choice of district i's representative.

4.2.1 Uniform Taxation

With uniform taxation and representatives of types λ_1 and λ_2, the optimal choice of district i's representative is

$$\left(g_1^i(\lambda_i), g_2^i(\lambda_i) \right) = \arg \max_{(g_i, g_{-i})} \left\{ \lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - \frac{p}{2}(g_i + g_{-i}) \right\}.$$

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[11] Given any two types $\hat{\lambda}_i$ and λ_i' such that $\lambda_i' < \hat{\lambda}_i < \lambda$ or $\lambda < \hat{\lambda}_i < \lambda_i'$, type λ citizens always prefer type $\hat{\lambda}_i$ citizens.

It is straightforward to verify that

$$(g_i^i(\lambda_i), g_{-i}^i(\lambda_i)) = \left(\frac{2\lambda_i(1-\kappa)}{p}, \frac{2\lambda_i\kappa}{p} \right), \quad i \in \{1,2\}.$$

The level of public goods spending depends only on the decisive representative's preference for public goods and the level of spillovers. The stronger the preferences for public goods of the decisive representative, the higher the spending. Furthermore, spending for the representative's domestic public good varies inversely with spillovers, while the other district's public good expenditures vary proportionally with spillovers.

When the representative types are λ_1 and λ_2, a citizen of type λ in region i obtains an expected public goods surplus of

$$\Delta U_{\lambda,i} = \frac{1}{2} \left\{ \lambda \left[(1-\kappa)\ln\frac{2\lambda_i(1-\kappa)}{p} + \kappa\ln\frac{2\lambda_i\kappa}{p} \right] - \lambda_i + \right.$$
$$\left. + \lambda \left[(1-\kappa)\ln\frac{2\lambda_{-i}\kappa}{p} + \kappa\ln\frac{2\lambda_{-i}(1-\kappa)}{p} \right] - \lambda_{-i} \right\}.$$

Again we assume that the representatives will be of the majority preferred types. A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district i the median type prefers λ_i^* to any other type $\lambda \in \langle 0, \lambda^{max} \rangle$, given the other district's representative type λ_{-i}^*.[12] This means that $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if it is a Nash equilibrium of the two-player game in which each player has strategy set $\langle 0, \lambda^{max} \rangle$ and player $i \in \{1,2\}$ has payoff function

$$\Delta U_{m_i}(\lambda_i) = \frac{1}{2} \left\{ m_i \left[(1-\kappa)\ln\frac{2\lambda_i(1-\kappa)}{p} + \kappa\ln\frac{2\lambda_i\kappa}{p} \right] - \lambda_i + \right.$$
$$\left. + m_i \left[(1-\kappa)\ln\frac{2\lambda_{-i}\kappa}{p} + \kappa\ln\frac{2\lambda_{-i}(1-\kappa)}{p} \right] - \lambda_{-i} \right\}.$$

Taking first-order conditions and solving yields

$$(\lambda_1^*, \lambda_2^*) = (m_1, m_2).$$

Thus, an elected pair of representatives will be of types (m_1, m_2) and will choose a policy which reflects their public goods preferences. So we have:

Lemma 2. *Suppose uniform taxation and centralization with a minimum winning coalition view of the legislature. Then, $(g_1, g_2) = (2m_1(1-\kappa)/p, 2m_1\kappa/p)$ with probability $1/2$ and $(g_1, g_2) = (2m_2\kappa/p, 2m_2(1-\kappa)/p)$ with probability $1/2$.*

This result illuminates the main drawbacks of centralization with a minimum winning coalition legislature and uniform taxation:

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[12] If citizens of type λ prefer a type $\hat{\lambda}_i$ candidate to a type λ_i' candidate, where $\hat{\lambda}_i < \lambda_i'$ ($\hat{\lambda}_i > \lambda_i'$), then so must all citizens of types lower (higher) than λ. This implies that a majority of citizens in district i prefer a type $\hat{\lambda}_i$ candidate to a type λ_i' candidate if and only if the median type prefers a type $\hat{\lambda}_i$ candidate to a type λ_i' candidate.

1. *Uncertainty*. Each district faces uncertainty as to the amount of public good that it will receive, reflecting the uncertainty in the identity of the minimum winning coalition.

2. *Misallocation*. Public expenditures across regions are skewed towards those inside the winning coalition.

4.2.2 Comparative Statics

The only situation in which centralization produces the surplus-maximizing level is when the districts are identical and spillovers are maximal ($\kappa = 1/2$). When districts differ ($m_1 > m_2$) and spillovers are complete, spending is allocated equally across regions but district 1's representative over-provides local public goods, while district 2's representative under-provides them. While higher levels of spillovers still lead those in the minimum winning coalition to allocate public goods to districts outside the coalition, it is only to the extent that this benefits those inside the coalition.

For low levels of spillovers, the misallocation problem is at its worse. Public goods are over-provided to regions in the minimum winning coalition and under-provided to those districts that are outside the coalition, reflecting the budgetary externality created by common financing. However, this drawback is significantly suppressed as long as the non-uniform tax system is introduced.

4.2.3 Non-Uniform Taxation

With non-uniform taxation and representatives of types λ_1 and λ_2, the optimal choice of region i's representative is

$$(g_1^i(\lambda_i), g_2^i(\lambda_i)) = \arg\max_{(g_i, g_{-i})} \{\lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - (pg_i(1-\kappa) + pg_{-i}\kappa)\}.$$

It is easily checked that

$$(g_1^i(\lambda_i), g_2^i(\lambda_i)) = \left(\frac{\lambda_i}{p}, \frac{\lambda_i}{p}\right), \quad i \in \{1, 2\}.$$

As above, if the representative types are λ_1 and λ_2, a citizen of type λ in district i obtains an expected public goods surplus of

$$\Delta U_{\lambda,i} = \frac{1}{2}\left\{\lambda\left[(1-\kappa)\ln\frac{\lambda_i}{p} + \kappa\ln\frac{\lambda_i}{p} + (1-\kappa)\ln\frac{\lambda_{-i}}{p} + \kappa\ln\frac{\lambda_{-i}}{p}\right] - \lambda_i - \lambda_{-i}\right\}.$$

Analogously to the case of uniform taxation, we arrive at the conclusion that an elected pair of representatives will be of types (m_1, m_2) and that they will choose a policy which reflects their preferences. This establishes:

Lemma 3. *Suppose non-uniform taxation and centralization with a minimum winning coalition view of the legislature. Then $(g_1, g_2) = (m_1/p, m_1/p)$ with probability $1/2$ and $(g_1, g_2) = (m_2/p, m_2/p)$ with probability $1/2$.*

Compared to the case of uniform taxation, the problem of *uncertainty* remains due to the unknown identity of the coalition. However, the drawback of *misallocation* is significantly reduced, reflecting the fact that each district is taxed according to its proportional consumption of both local public goods. This suppresses the incentives of the coalition members to allocate too much of the public goods to their districts while forgetting about the regions outside the coalition.

4.2.4 Comparative Statics

The levels of public goods are independent of spillovers. They depend only on the preferences of the decisive representative which then chooses uniform provision of public goods. With identical representatives, centralization with non-uniform taxation produces the surplus-maximizing levels of local public goods. When m_1 exceeds m_2 and spillovers are complete, region 1's representative over-provides local public goods, while district 2's representative under-provides them.

The misallocation problem is at its worse when the spillovers are lower than complete. The levels of public goods provided are further from the optimal, aggregate surplus enhancing levels. However, the extent of these misallocations is lower than that under the centralized system with uniform taxation.

4.3 Centralization versus Decentralization
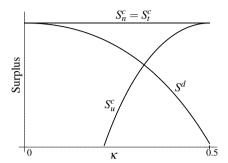
4.3.1 Homogeneous Districts

Decentralization produces the surplus-maximizing public goods levels if and only if spillovers do not occur. We have already seen that public goods levels under *centralization with uniform taxation* are surplus maximizing when the spillovers are complete and the districts are homogeneous. It follows that, in the case of identical districts, decentralization dominates when the spillovers are small and centralization is preferred when the spillovers are large.

Centralization with non-uniform taxation produces the surplus-maximizing public goods levels when the districts are identical. This surplus is independent of spillovers and is higher than the surplus under decentralization for all κ except when the spillovers are absent. In such a case, both systems generate the surplus-maximizing public goods levels. The next proposition and *Figure 3* summarize these results.

Proposition 2. *Suppose that the assumptions of the political economy analysis are satisfied, the centralized decision making relies on the minimum winning coalition, and the districts are identical. Then:*

(i) *If the taxation is uniform, there is a critical value of κ, strictly greater than 0 but less than $1/2$, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level.*

(ii) *If the taxation is non-uniform across districts and spillovers are present ($\kappa > 0$), a centralized system produces a higher level of surplus than does decentralization. In the absence of spillovers ($\kappa = 0$), the two systems generate the same level of surplus.*

FIGURE 3   Aggregate Public Goods Surpluses under Decentralization (S^d), Centralization in the Standard Analysis (S^c_t) and Centralization with Uniform (S^c_u) and Non-Uniform (S^c_n) Taxation in the Case of *Identical Districts*

(iii) The surplus under centralization with non-uniform taxation equals that under the centralized system in the standard analysis for all levels of spillovers. These surpluses are higher than that under centralization with uniform taxation except when the spillovers are maximal ($\kappa = 1/2$). In such a case, all three systems of centralization produce the same public goods surplus.

There are two comparisons which require analysis. First, comparing Proposition 2 with its counterpart in part (i) of Proposition 1, there is one significant difference. With identical districts, the centralized system in the standard analysis is supposed to dominate decentralization for all $\kappa > 0$. However, centralization based on the minimum winning coalition and uniform taxation no longer dominates for low levels of spillovers, as those inside the coalition have low incentives to provide public goods to the outside regions. This is further combined with the uncertain identity of the coalition. With higher spillovers, uncertainty remains but the decisive representatives have higher incentives to provide more public goods to both districts, which increases the surplus under centralization. Thus, political economy analysis weakens the case for centralization when taxation is uniform.

Second, a comparison of the two centralized systems under the political economy analysis generates a strong case for centralization with non-uniform taxation, which dominates for all $\kappa < 1/2$. This is due to the effects that each taxation has on the decisions about the allocation of public expenditures. When the taxation is uniform, each district pays the same head tax independent of the level of public goods received. This motivates coalition members to allocate as much as they like to their districts. In contrast, under the centralized system with non-uniform taxation there is no such effect, as each district is taxed according to its proportional consumption of both local public goods. This balances the allocated levels and centralization with non-uniform taxation significantly dominates centralization with a uniform tax system. Furthermore, the resulting surplus under centralization with non-uniform taxation is the same as under centralization in the standard analysis.[13]

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[13] This is due to the fact that the provision of public goods and the actual taxes under centralization with non-uniform taxation are the same as under the centralized system in the standard analysis. With identical

4.3.2 Heterogeneous Districts

When the regions are heterogeneous, *a centralized system with uniform taxation* still dominates decentralization for high levels of spillovers and its performance is increasing in spillovers. Thus, there is a critical value of κ under which decentralization is preferred and above which centralization dominates.

Centralization with non-uniform taxation is independent of spillovers, produces a higher level of surplus than decentralization for maximal spillovers and a lower level of surplus for zero spillovers. It follows that there exists a critical value of κ above which the centralized system dominates and under which decentralization is preferred. However, this critical value is lower than that in the uniform taxation case. Again, the following proposition and *Figure 4* summarize these findings.

Proposition 3. *Suppose that the assumptions of the political economy analysis are satisfied, the centralized decision-making relies on the minimum winning coalition, and the districts are non-identical. Then*
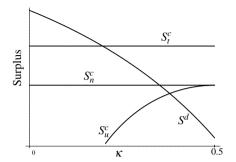
(i) *If the taxation is uniform, there is a critical value of κ, strictly greater than 0 but less than $1/2$, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level. This critical level is higher than that in the standard analysis.*

(ii) *If the taxation is non-uniform across districts, there is a critical value of κ, strictly greater than 0 but less than $1/2$, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level. This critical level is higher than that in the standard analysis and lower than that under centralization with uniform taxation.*

(iii) *The surplus under centralization in the standard analysis is higher than the surpluses under both centralized systems in the political economy analysis for all levels of spillovers. Furthermore, the surplus under centralization with non-uniform taxation is higher than that under the centralized system with uniform taxation except when the spillovers are maximal $(\kappa = 1/2)$. In such a case, the two systems produce the same public goods surplus.*

As above, two juxtapositions can be observed. First, comparing Proposition 3 with its relevant counterpart in part (ii) of Proposition 1 reveals that centralization with a non-cooperative legislature creates an even larger incongruity when the districts are heterogeneous. This exacerbated misallocation problem combined with the persistent drawback of uncertainty results in a weakened case for centralization compared with the centralized system in the standard analysis. However, the fundamental qualitative conclusions remain unchanged under the political economy analysis as under the traditional one: for low spillover levels, decentralization dominates; when the spillovers are high, centralization is preferred.[14]

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districts, each representative would choose such uniform public goods levels that would be chosen by all the other representatives, if elected.

[14] What happens with both critical levels of spillovers as heterogeneity increases may be analyzed by letting $S^d(\kappa, \alpha), S_u^c(\kappa, \alpha)$ and $S_n^c(\kappa, \alpha)$ denote surpluses under decentralization and under centralization with

FIGURE 4   Aggregate Public Goods Surpluses under Decentralization (S^d), Centralization in the Standard Analysis (S^c_t) and Centralization with Uniform (S^c_u) and Non-Uniform (S^c_n) Taxation in the Case of *Non-Identical Districts*

Second, the comparison of the two centralized systems with different taxations remains as above. This is because the two systems' surpluses decrease at the same rate with increasing heterogeneity.[15] Thus, centralization with non-uniform taxation dominates centralization with a uniform tax system for all $\kappa < 1/2$. When spillovers are complete, the two systems generate the same level of surplus. This reflects the fact that, in the uniform taxation case, a representative in the winning coalition has incentives to provide the same level of public goods to both regions, which corresponds to the case of non-uniform taxation.

Furthermore, the surplus under centralization with non-uniform taxation is lower than that under centralization in the standard analysis. In the non-uniform taxation case, increasing heterogeneity causes the potential provisions of the two representatives to vary still more. This decreases the surplus under centralization with non-uniform taxation, and because the surplus under the centralized system in the traditional analysis is independent of heterogeneity, centralization with a non-uniform tax system gener-

uniform and non-uniform taxation, respectively, when $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$, where $\alpha \in \langle 1/2, 1 \rangle$ is the degree of heterogeneity between the regions. The first critical level of κ, denoted $\kappa^*_1(\alpha)$, is uniquely defined by the equation $S^d(\kappa^*_1, \alpha) = S^c_u(\kappa^*_1, \alpha)$. To show that κ^*_1 is an increasing function of α, it is necessary to show that for all $\alpha \in (1/2, 1)$,

$$\frac{\partial S^d(\kappa^*_1, \alpha)}{\partial \alpha} - \frac{\partial S^c_u(\kappa^*_1, \alpha)}{\partial \alpha} > 0.$$

Differentiating, we obtain

$$\frac{\partial S^d(\kappa, \alpha)}{\partial \alpha} - \frac{\partial S^c_u(\kappa, \alpha)}{\partial \alpha} = \underbrace{\omega \left(\frac{1}{2} - \kappa \right)}_{>0} \left(2\ln\frac{\alpha}{1-\alpha} - \frac{1-2\alpha}{\alpha(1-\alpha)} \right).$$

The expression in the latter parentheses equals zero when $\alpha = 1/2$ and is positive for all α in the range $(1/2, 1)$. Thus, the difference is positive for all $\alpha \in (1/2, 1)$ and $\kappa^*_1 < 1/2$, which implies that the critical level of spillovers increases with increasing heterogeneity. The second critical level of κ, denoted $\kappa^*_2(\alpha)$, is uniquely defined by the equation $S^d(\kappa^*_2, \alpha) = S^c_n(\kappa^*_2, \alpha)$. Due to the fact that $\partial S^c_u(\kappa, \alpha)/\partial \alpha = \partial S^c_n(\kappa, \alpha)/\partial \alpha$ for all κ and α, the critical level of spillovers increases with increasing heterogeneity for the non-uniform taxation case as well.

[15] See the previous footnote.

ates a lower public goods surplus.[16] It follows that political economy analysis weakens the case for centralization when the taxation is non-uniform but not as considerably as in the case of a centralized system with uniform taxation.

5. Cooperative Centralization and Two Forms of Taxation

Under the minimum winning coalition view of legislative decision-making, policy outcomes are ex ante Pareto inefficient from the viewpoint of the representatives. Thus, legislators may find a way around the inefficiency created by majoritarian decision-making criteria and prefer a less random outcome to the "feast or famine" implied by the minimum winning coalition theory. The representatives with power may, to a given extent, allocate benefits to those outside the coalition on the understanding that non-members would behave similarly if they were in power. However, there are many pairs of local public goods levels that are efficient from the viewpoint of the representatives and that ex ante Pareto dominate the minimum winning coalition outcomes.

Here we will assume the case where the representatives agree to the public goods allocation that maximizes their joint surplus, i.e. their behavior can be described by the *utilitarian bargaining solution*. This means that each representative now maximizes the same utility function as the others. They agree to form a coalition where everybody will have a weight in the decision-making process, not just those who succeed in forming a minimum winning coalition. This norm requires representatives to take into account the costs and benefits to their colleagues and would seem to offer centralization the best chance of dominating decentralization given our welfare criterion. But the extent to which centralization will dominate decentralization will again depend on the form of taxation.

5.1 Uniform Taxation

With uniform taxation and representatives of types λ_1 and λ_2, the policy outcome written as $g_1(\lambda_1, \lambda_2), g_2(\lambda_1, \lambda_2)$ will now maximize the representatives' joint surplus given by

$$\sum_{i=1}^{2} \{\Delta U_{\lambda_i}\} = \sum_{i=1}^{2} \left\{ \lambda_i[(1-\kappa)\ln g_i + \kappa \ln g_{-i}] - \frac{p}{2}(g_i + g_{-i}) \right\}.$$

It is straightforward to show that the public goods levels maximizing this joint surplus are

$$(g_1(\lambda_1, \lambda_2), g_2(\lambda_1, \lambda_2)) = \left(\frac{\lambda_1(1-\kappa) + \lambda_2 \kappa}{p}, \frac{\lambda_1 \kappa + \lambda_2(1-\kappa)}{p} \right).$$

It is clear that if both districts elected representatives of the median types, the legislature would select the surplus-maximizing public goods levels.

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[16] Let $S_t^c(\kappa, \alpha)$ and $S_n^c(\kappa, \alpha)$ denote the surpluses under centralization in the standard analysis and with non-uniform taxation, respectively, when $(m_1, m_2) = (\alpha \omega, (1-\alpha)\omega)$, where $\alpha \in (1/2, 1)$ measures the degree of heterogeneity. From the previous discussion we know that S_t^c is independent of heterogeneity while $\partial S_n^c(\kappa, \alpha)/\partial \alpha = \omega(1-2\alpha)/2\alpha(1-\alpha) < 0$ for all $\alpha \in (1/2, 1)$. This implies that increasing heterogeneity decreases the surplus under centralization with a non-uniform tax system, which is then lower than that under a centralized system in the traditional analysis.

If the representative types are λ_1 and λ_2, a citizen of type λ in district i obtains a public goods surplus of

$$\Delta U_{\lambda,i} = \lambda \left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)+\lambda_{-i}\kappa}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)+\lambda_i\kappa}{p} \right] - \frac{\lambda_1+\lambda_2}{2}.$$

Turning to the election stage, we again assume that the pair of representatives will be of the majority preferred types defined in the by now familiar way. The main additional complication created by a cooperative legislature lies in finding the majority preferred types. This is because the public goods level for each region depends on the type of legislator in both districts and, thereby, generates incentives for citizens in each district to delegate policy-making strategically to a representative with different tastes than their own. This intention arises because sincere voting becomes suboptimal now.

To begin with, note that a pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district i the median type prefers λ_i^* to any other type $\lambda \in \langle 0, \lambda^{max} \rangle$, given the other district's type λ_{-i}^*.[17] Thus, $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if it is a Nash equilibrium of the two player game in which each player has strategy set $\langle 0, \lambda^{max} \rangle$ and player $i \in \{1,2\}$ has payoff function

$$U_i(\lambda_1, \lambda_2) = m_i \left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)+\lambda_{-i}\kappa}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)+\lambda_i\kappa}{p} \right] - \frac{\lambda_1+\lambda_2}{2}.$$

In this game, the district i median citizen tries to manipulate λ_i so that he obtains something close to his preferred policy outcome anticipating the election outcomes in the other region and the subsequent working of the legislature.[18] He only has one degree of freedom, λ_i, but two objectives, (g_1, g_2). While raising λ_i always leads to an increase in g_i, if $\kappa > 0$ it also raises g_{-i}.

To state the equilibria, define $\hat{\kappa}$ as the solution to

$$\frac{m_1}{m_2} = \frac{\hat{\kappa}^3+(1-\hat{\kappa})^3}{\hat{\kappa}(1-\hat{\kappa})}.$$

When the districts are identical, $\hat{\kappa} = 1/2$. In the non-identical districts case, $\hat{\kappa} < 1/2$. Then:

Lemma 4. *Suppose uniform taxation in cooperative centralization. If $\kappa < \hat{\kappa}$,*

$$(g_1, g_2) = \left(\frac{2m_1[(1-\kappa)^4-k^4]}{\left[(1-\kappa)^2-\dfrac{m_1}{m_2}\kappa^2\right]p}, \frac{2m_1[(1-\kappa)^4-k^4]}{\left[\dfrac{m_1}{m_2}(1-\kappa)^2-\kappa^2\right]p} \right)$$

and if $\kappa \geq \hat{\kappa}$,

$$(g_1, g_2) = \left(\frac{2m_1(1-\kappa)}{p}, \frac{2m_1\kappa}{p} \right).$$

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[17] If district i elects a citizen of a higher type, then it receives more of both public goods. Then the same argument applies as in footnote 17.

[18] To put it more rigorously, all citizens in region i now have an interest in manipulating λ_i to obtain something close to their preferred policy outcome. In other words, all voters in district i have the same interest in shifting λ_i according to their preferences and expectations of the election outcomes in the other regions and subsequent working of the legislature.

It can be easily seen that the cooperative legislature does not select the surplus-maximizing public goods levels. While a cooperative legislature deals with problems of uncertainty and misallocation that were present in the non-cooperative legislature, *strategic delegation* emerges: each district's median voter delegates policy-making to a representative of different than median type.

5.1.1 Comparative Statics

When the regions are identical ($m_1 = m_2 = m$), it follows from Lemma 4 that $g_1 = g_2 = g$, and $g = 2m[(1 - \kappa)^2 + \kappa^2]/p$. Recall that with identical districts, the surplus-maximizing level of public goods is $g_1 = g_2 = m/p$. Thus, local public goods are over-provided in both regions for all $\kappa < 1/2$. the extent of this over-provision decreases with increasing spillovers and over-provision does not occur only when the spillovers are maximal ($\kappa = 1/2$). In such a case, local public goods are provided optimally.

The incentives to strategically delegate can be seen most clearly in the case of zero spillovers. Then, the optimal spending levels for the median voter from region 2 are $(g_1, g_2) = (0, 2m/p)$. Assume for a moment that both districts elect median type representatives. This would lead to policy outcome $(g_1, g_2) = (m/p, m/p)$. But if district 2 elected a representative with a stronger taste for public spending, it would get more of its local public goods with no impact on district 1's public goods level. Thus, each region is drawn to elect a type $2m$ representative.

As spillovers increase, the optimal spending levels in the two districts for each median voter converge. Electing a representative with a higher preference for public goods spending increases spending in the other region as well. Thus, the districts elect representatives with preferences closer to their median. When the spillovers are maximal, each region elects a median type representative and local public goods are provided at the surplus-maximizing level.

With heterogeneity, an additional conflict over the *level* of public spending enters the picture, which can be seen most clearly in the case of complete spillovers. If $\kappa = 1/2$ and each region elects a representative of the median type, the public goods levels are $g_1 = g_2 = (m_1 + m_2)/2p$. This common level is too low for district 1's median voter and too high for region 2's. This gives district 1's median voter an incentive to have a higher representative type to boost public goods spending, while region 2's median voter desires a representative with lower public goods preferences. They pull in opposite directions until one or both districts has put in their most extreme type. Our assumption that $2m_1 < \lambda^{max}$ implies that district 1 can obtain its preferred public goods level when district 2 has put in its most extreme type. Thus, district 1's median voter ends up getting his preferred outcome of $g_1 = g_2 = m_1/p$.

This additional conflict of interest creates a complex relationship between spillovers and public goods levels. Analyzing the solutions described in the Lemma, it can be shown that district 1's public goods level is decreasing in the level of spillovers for sufficiently small κ and $\kappa > \hat{\kappa}$.[19] However, it is increasing in spillovers for κ sufficiently close to but less than $\hat{\kappa}$. This reflects the conflict over spending levels that arises as spillovers increase. To prevent district 2 from pulling down spending in both districts, district 1's median voter elects a representative with a higher public goods valuation, raising district 1's public goods level. Region 2's public goods level is decreasing in

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[19] This and the other claims concerning the public goods levels in Lemma 4 have been established in Besley and Coate (1999, 2000). Here we add analysis of non-uniform taxation.

spillovers for $\kappa < \hat{\kappa}$ and increasing thereafter. It increases for spillover levels in excess of $\hat{\kappa}$, because it is now effectively controlled by district 1's median voter.

Comparing these outcomes with the surplus-maximizing levels of public goods, district 1's public goods level is always too high. The level provided to region 2 is too high for small κ and when κ is sufficiently large. However, it is less than the surplus-maximizing level for κ sufficiently close to $\hat{\kappa}$. Note that this under-provision is in contrast to the over-provision results for the case of identical districts.

It is clear at this point that, although the legislature follows the utilitarian bargaining solution, the problem of strategic delegation causes that this solution may still be far from the surplus-maximizing ideal. By introducing non-uniform taxation as defined above, we will nevertheless show that this problem is significantly suppressed.

5.2 Non-Uniform Taxation

If the taxation is non-uniform and the representatives are of types λ_1 and λ_2, the policy outcome $(g_1(\lambda_1,\lambda_2), g_2(\lambda_1,\lambda_2))$ will again maximize the representatives' joint surplus given by

$$\sum_{i=1}^{2}\{\Delta U_{\lambda_i}\} = \sum_{i=1}^{2}\{\lambda_i[(1-\kappa)\ln g_i + \kappa\ln g_{-i}] - p[(1-\kappa)g_i + \kappa g_{-i}]\}.$$

It is straightforward to verify that the public goods levels maximizing this joint surplus are again

$$(g_1(\lambda_1,\lambda_2), g_2(\lambda_1,\lambda_2)) = \left(\frac{\lambda_1(1-\kappa)+\lambda_2\kappa}{p}, \frac{\lambda_1\kappa+\lambda_2(1-\kappa)}{p}\right).$$

Thus, as applicable also in the uniform taxation case, if both regions elected representatives of the median types, the legislature would select the surplus-maximizing levels of public goods.

If the representatives are of types λ_1 and λ_2, a citizen of type λ in region i obtains public goods surplus

$$\Delta U_{\lambda,i} = \lambda\left[(1-\kappa)\ln\frac{\lambda_i(1-\kappa)+\lambda_{-i}\kappa}{p} + \kappa\ln\frac{\lambda_{-i}(1-\kappa)+\lambda_i\kappa}{p}\right] - \\ - [\lambda_i(1-2\kappa+2\kappa^2)+\lambda_{-i}(2\kappa-2\kappa^2)].$$

As was the case in the previous section, the main complication lies in finding the majority preferred types when sincere voting is suboptimal. This complication is again due to the fact that the public goods level in each district depends on the type of legislator in both regions and, thereby, generates incentives for citizens in each region to strategically delegate policy-making to a representative with different public goods preferences than their own.

A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district i the median type prefers λ_i^* to any other type $\lambda \in \langle 0, \lambda^{max}\rangle$, given the other region's type λ_{-i}^*.[20] Thus, $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if it is a Nash

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[20] If district i elects a citizen of a higher (more public-good loving) type, then it receives more of both public goods. Then the same argument applies as in footnote 17.

equilibrium of the two player game in which each player has strategy set $\langle 0, \lambda^{max} \rangle$ and player $i \in \{1, 2\}$ has payoff function

$$U_i(\lambda_1, \lambda_2) = m_i \left[(1-\kappa) \ln \frac{\lambda_i(1-\kappa) + \lambda_{-i}\kappa}{p} + \kappa \ln \frac{\lambda_{-i}(1-\kappa) + \lambda_i \kappa}{p} \right]$$
$$- [\lambda_i(1 - 2\kappa + 2\kappa^2) + \lambda_{-i}(2\kappa - 2\kappa^2)].$$

In this game, the district i median citizen tries to manipulate λ_i so that he obtains something close to his preferred policy outcome anticipating the election outcomes in the other region and the subsequent working of the legislature. While raising λ_i always leads to an increase in g_i, if $\kappa > 0$ it also raises g_{-i}.

To state the equilibria, define $\hat{\kappa}$ as the solution to

$$\frac{m_1}{m_2} = \frac{\hat{\kappa}^3 + (1-\hat{\kappa})^3}{\hat{\kappa}(1-\hat{\kappa})}.$$

When the districts are identical, $\hat{\kappa} = 1/2$. In the non-identical districts case, $\hat{\kappa} < 1/2$. Then we have:

Lemma 5. *Suppose non-uniform taxation and a cooperative legislature. If $\kappa < \hat{\kappa}$,*

$$(g_1, g_2) = \left(\frac{m_1[(1-\kappa)^2 - k^2]}{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2 \right]p}, \frac{m_1[(1-\kappa)^2 - k^2]}{\left[\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2 \right]p} \right)$$

and if $\kappa \geq \hat{\kappa}$,

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)}{[(1-\kappa)^2 + \kappa^2]p}, \frac{m_1 \kappa}{[(1-\kappa)^2 + \kappa^2]p} \right).$$

It is easily seen that the cooperative legislature does not always select the surplus-maximizing public good levels. However, although *strategic delegation* occurs also when the taxation is non-uniform, it is significantly suppressed compared to the uniform taxation case in a way we will now explain.

5.2.1 Comparative Statics

With identical districts ($m_1 = m_2 = m$), the Lemma implies that $(g_1, g_2) = (m/p, m/p)$, which are the surplus maximizing public goods levels. Thus, strategic delegation is *completely eliminated* when the taxation is non-uniform and the districts are homogeneous. This is because neither district is drawn to elect a representative with a stronger taste for public goods, as each region knows that it will have to pay proportionally to its consumption. If it elected a higher type representative, the resulting increase in the provision of local public goods would be fully financed by the given district, which is in contrast to the uniform-taxation case, where both districts participate in this increase in financing by only a half. Therefore, each district elects a median type representative and the local public goods are provided optimally, regardless of the level of spillovers.

Heterogeneity again gives rise to strategic delegation, but to a lesser extent compared to the uniform taxation case. The only situation in which strategic delegation with non-uniform taxation is as strong as in the case of uniform taxation occurs when the spillovers are maximal. If $\kappa = 1/2$ and each district elects a median type representative, the policy outcome is $g_1 = g_2 = (m_1 + m_2)/2p$. But this level is too low for region 1's median voter and too high for that of region 2.[21] This gives district 1's median voter an incentive to elect a higher type representative and region 2's median voter an incentive to have a lower representative type. So they pull in opposite directions until one or both districts has put in their most extreme type. Under our assumption that $2m_1 < \lambda^{max}$, region 1 can obtain its preferred public goods levels when district 2 has put in its most extreme type.

The relationship between public goods levels and spillovers is again very complex. District 1's public goods level is increasing in the level of spillovers for $\kappa < \hat{\kappa}$.[22] This appears puzzling, as district 1's median voter's preferred public goods level is actually constant in spillovers. The result reflects the conflict over spending levels. To prevent district 2 from pulling down spending in both regions, district 1's median voter elects a representative with a higher taste for public spending, raising region 1's public goods level. Furthermore, district 1's public goods level is decreasing for κ sufficiently close to $1/2$. However, it can increase or decrease for κ sufficiently close to but higher than $\hat{\kappa}$. District 2's public goods level is decreasing in the level of spillovers for $\kappa < \hat{\kappa}$ and increasing thereafter. It increases for spillovers in excess of $\hat{\kappa}$ because it is now effectively controlled by region 1's median voter.

Two comparisons require analysis here. Firstly, comparing these policy outcomes with the surplus-maximizing levels, district 1's public goods level is too high for all levels of spillovers except when $\kappa = 0$. In this case, region 1's public good is provided at the surplus-maximizing level. The level provided to district 2 is too low for all $\kappa < \hat{\kappa}$ and for κ higher than but sufficiently close to $\hat{\kappa}$. the only exception here is again when $\kappa = 0$. In this case district 2's public goods level is the surplus-maximizing one. Moreover, district 2's public goods level is too high for κ sufficiently close to $1/2$.

Secondly, comparing public goods levels in the two tax systems, each district's public goods are provided at higher level when the taxation is uniform than under the non-uniform tax system, except when spillovers are maximal. In such a case, both systems generate the same public goods levels. Thus, non-uniform taxation suppresses, though does not completely eliminate, the incentives to delegate policy-making strategically to representatives with higher preferences for public spending.

5.3 Centralization versus Decentralization

5.3.1 Homogeneous Districts

We already know that decentralization produces the surplus-maximizing public goods levels only in the case of zero spillovers. Public goods levels under *centralization with uniform taxation* are surplus maximizing only when spillovers are complete and the districts are identical. It follows that, in the case of identical regions, decentralization dominates when spillovers are small and centralization is preferred when

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[21] The optimal spending levels for district 1's median voter are $g_1 = g_2 = m_1/p$, whereas for region 2's median voter they are $g_1 = g_2 = m_2/p$.

[22] This and the other claims concerning the public goods levels from Lemma 5 are established in the appendix.
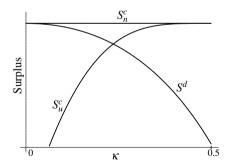
spillovers are large. the surplus under centralization with uniform taxation increases with increasing κ and a critical value of spillovers exists above which centralization is welfare superior.

When the districts are identical, *centralization with non-uniform taxation* produces the surplus maximizing public goods levels for all spillover levels. This surplus is higher than that under decentralization for all κ except when the spillovers are absent. In such a case, both systems generate the surplus-maximizing public goods levels. The next proposition and *Figure 5* summarize these results.

Proposition 4. *For cooperative legislature and identical districts:*

(i) *If the taxation is uniform, there is a critical value of κ, strictly greater than 0 but less than $1/2$, such that a centralized system produces a higher level of surplus if and only if κ exceeds this critical level.*

(ii) *If the taxation is non-uniform across districts and spillovers are present $(\kappa > 0)$, a centralized system produces a higher level of surplus than does decentralization. In the absence of spillovers $(\kappa = 0)$, the two systems generate the same level of surplus.*

(iii) *Surplus under centralization with non-uniform taxation is higher than that under centralization with uniform taxation except when the spillovers are maximal $(\kappa = 1/2)$. In such a case, both systems produce the same public goods surplus.*

FIGURE 5 Aggregate Public Goods Surpluses under Decentralization (S^d), Centralization with Uniform (S_u^c) and Non-Uniform (S_n^c) Taxation for *Identical Districts*

There are two important findings which require analysis. First, when the taxation is uniform, decentralization dominates when spillovers are low and centralization is preferred when spillovers are high, whereas a critical level of spillovers exists and is in the range $(0, 1/2)$. This is in line with the results obtained in the preceding sections when the legislature was based on the minimum-winning coalition.

Second, this does not hold, however, when the taxation under centralization is non-uniform. Such a system produces surplus-maximizing public goods levels regardless of

the level of spillovers and dominates decentralization for all $\kappa > 0$. Thus, non-uniform taxation is a significant tool for eliminating strategic delegation in the case of identical regions.

5.3.2 Heterogeneous Districts

When the districts are heterogeneous, decentralization continues to dominate *centralization with uniform taxation* when spillovers are small and centralization is preferred when spillovers are large. The case of *centralization with non-uniform taxation* is a bit more complicated when it comes to heterogeneous districts. Centralization still dominates decentralization when spillovers are large, but it dominates decentralization even when spillovers are small. Furthermore, it may be that centralization with non-uniform taxation produces a higher public goods surplus than does decentralization for all $\kappa > 0$. However, there is no general presumption that this is always so. Decentralization may dominate centralization when κ is sufficiently close to $\hat{\kappa}$.

These findings are summarized in the following proposition and *Figures 6* and *7*.

Proposition 5. *For cooperative legislature and non-identical districts:*

(i) *If the taxation is uniform, a decentralized system produces a higher level of surplus when spillovers are sufficiently small, while a centralized system produces a higher level of surplus when spillovers are sufficiently large.*

(ii) *If the taxation is non-uniform, a centralized system produces a higher level of surplus than does decentralization when spillovers are sufficiently large and when spillovers are sufficiently small but positive. In the absence of spillovers, the two systems generate the same public goods surplus.*

(iii) *Surplus under centralization with non-uniform taxation is higher than that under centralization with a uniform tax system except when the spillovers are maximal ($\kappa = 1/2$). In this case, both systems produce the same public goods surplus.*

FIGURE 6   Aggregate Public Goods Surpluses under Decentralization (S^d), Centralization with Uniform (S_u^c) and Non-Uniform (S_n^c) Taxation for *Non-Identical Districts* and *High Heterogeneity*
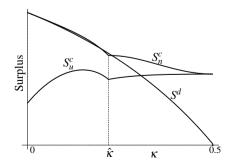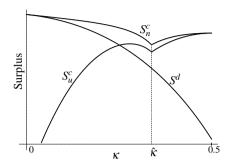
FIGURE 7 Aggregate Public Goods Surpluses under Decentralization (S^d), Centralization with Uniform (S^c_u) and Non-Uniform (S^c_n) Taxation for *Non-Identical Districts* and *Low Heterogeneity*

Three important lessons can be drawn from these statements. Firstly, the basic conclusions of part (i) of Proposition 3 generalize to the case of a cooperative legislature. When the centralized system is financed by uniform taxation, decentralization dominates centralization for low spillover levels, while centralization dominates for high levels of spillovers. The only difference here is that there need not exist a critical level of spillovers at all. This reflects the fact that there is no general presumption that the relative performance of centralization is always increasing in spillovers. Surplus under centralization is decreasing in κ for κ sufficiently close to but lower than $\hat{\kappa}$.

Secondly, the conclusions just mentioned do not carry over to centralization with a non-uniform tax system. Under this taxation, the centralized system produces a higher surplus than does decentralization even for low levels of spillovers. This is due to the nature of a non-uniform tax system, which means that the financing of public goods is not shared any more but is proportionally distributed between regions.

However, we cannot show that centralization always dominates decentralization for all spillover levels. This reflects the fact that when the districts are sufficiently heterogeneous, the decentralized system produces a higher surplus for κ sufficiently close to $\hat{\kappa}$, which is demonstrated in *Figure 6*.

On the other hand, when the regions do not differ very much in their public goods preferences, centralization with non-uniform taxation dominates decentralization for all $\kappa > 0$ (*Figure 7*). Thus, although strategic delegation does arise under this system as well, it does so to a much lesser extent compared to the uniform taxation case. However, the policy outcomes produced under this system can still be improved in the direction towards the surplus-maximizing ideal.

Finally, comparing the two tax systems, we must again conclude that centralization with non-uniform taxation dominates centralization with a uniform tax system for all $\kappa < 1/2$. When the spillovers are maximal, the two systems produce the same public goods surplus. This is in line with the results obtained for centralization based on the minimum winning coalition.

We have thus generalized the conclusion that a non-uniform tax system produces strictly better policy outcomes than does uniform taxation. In the case of a cooperative legislature, centralization based on this taxation may even dominate decentralization for all $\kappa > 0$, which is a stunning result.

6. Conclusions

This paper has taken a fresh look at the relative merits of centralized and decentralized provision of local public goods, closely following Besley and Coate (2003). It shows that allowing for non-uniform public goods and district-specific taxes means enhancing the performance of centralization relative to the case in which taxation and/or provision is uniform across regions. Flexibility in cost shares has a very positive effect on the performance of a centralized system in all of the studied cases. Specifically, a centralized system with non-uniform taxation appears to weakly dominate centralization with a uniform tax system for all levels of spillovers except when the spillovers are maximal. This result holds regardless of heterogeneity in tastes and the political economy assumptions.

When decisions are made by a legislature of locally elected representatives, a non-uniform tax system suppresses or completely eliminates the drawbacks created by a centralized system with uniform financing. If decisions on local public goods are made by a minimum winning coalition of representatives, non-uniform taxation significantly reduces (when the regions are non-identical) or completely eliminates (in the case of identical districts) the misallocation problem. Nevertheless, the uncertainty remains due to the unknown identity of the coalition in either case. If decisions are made on a more cooperative basis, then strategic delegation is significantly suppressed (when the districts are non-identical) or completely eliminated (in the case of identical regions).

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Appendix: Proofs

Proof of Proposition 1. The proofs of Propositions 1, 2, and 3 are not very difficult. Therefore, we omit them due to space constraints and provide only Proofs 4 and 5 with the associated Lemmas. However, they are available upon request from the authors.

Proof of Proposition 2. Available upon request.

Proof of Proposition 3. Available upon request.

Proof of Lemma 4. Due to the close similarity between the proof of this Lemma and Lemma 5 and the fact that in this paper we focus more on the non-uniform tax system, we refer to Besley and Coate (2003) for a thorough proof of this Lemma.

Proof of Lemma 5. As mentioned in the text, $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if $(\lambda_1^*, \lambda_2^*)$ is a Nash equilibrium of the two player game in which each player has strategy set $\langle 0, \lambda^{max} \rangle$ and player $i \in \{1, 2\}$ has payoff function $U_i(\lambda_1, \lambda_2)$. We prove the Lemma by calculating the set of equilibria of this game and computing the associated policy outcomes.

Note first that each player's payoff function is a twice continuously differentiable and strictly concave function of his strategy and each player's strategy set is compact and convex. Thus, the set of equilibria is non-empty. Moreover, $\partial^2 U_1 / \partial \lambda_1 \partial \lambda_2 < 0$ and $\partial^2 U_2 / \partial \lambda_2 \partial \lambda_1 < 0$, implying that types are strategic substitutes.

For $i = 1, 2$, let $r_i : \langle 0, \lambda^{max} \rangle \to \langle 0, \lambda^{max} \rangle$ denote the region i median voter's *reaction function*. By definition, for all $\lambda_2 \in \langle 0, \lambda^{max} \rangle$,

$$r_1(\lambda_2) = \arg\max\{U_1(r_1, \lambda_2) : r_1 \in \langle 0, \lambda^{max} \rangle\},$$

and for all $\lambda_1 \in \langle 0, \lambda^{max} \rangle$,

$$r_2(\lambda_1) = \arg\max\{U_2(\lambda_1, r_2) : r_2 \in \langle 0, \lambda^{max} \rangle\}.$$

Then, $(\lambda_1^*, \lambda_2^*)$ is an equilibrium of the game if and only if $(\lambda_1^*, \lambda_2^*) = (r_1(\lambda_2^*), r_2(\lambda_1^*))$.

Several general features of the reaction functions follow from the properties of the payoff functions. The fact that each player's payoff is a strictly concave and differentiable function of his strategy implies (i) that $r_1(\lambda_2) = 0$ if $\partial U_1(0, \lambda_2) / \partial \lambda_1 < 0$; (ii)

that $r_1(\lambda_2) = \lambda^{max}$ if $\partial U_1(\lambda^{max}, \lambda_2)/\partial \lambda_1 > 0$; and (iii) that otherwise $r_1(\lambda_2)$ is implicitly defined by the first-order condition $\partial U_1(r_1(\lambda_2), \lambda_2)/\partial \lambda_1 = 0$. In addition, the fact that types are strategic substitutes implies that $r_1(\lambda_2)$ is non-increasing. Analogous remarks apply to the district 2 median voter's reaction function.

It remains therefore to determine the details of each player's reaction function. Let $\lambda_2^{max}(\lambda_1^{max})$ denote the level of $\lambda_2(\lambda_1)$ beyond which district 1's median voter (district 2's median voter) would like a type 0 representative. These levels are implicitly defined by the equalities

$$\partial U_1(0, \lambda_2^{max})/\partial \lambda_1 = 0,$$

and

$$\partial U_2(\lambda_1^{max}, 0)/\partial \lambda_2 = 0.$$

Using the facts that

$$\frac{\partial U_1}{\partial \lambda_1} = m_1 \left[\frac{(1-\kappa)^2}{\lambda_1(1-\kappa) + \lambda_2 \kappa} + \frac{\kappa^2}{\lambda_2(1-\kappa) + \lambda_1 \kappa} \right] - (1 - 2\kappa + 2\kappa^2),$$

and

$$\frac{\partial U_2}{\partial \lambda_2} = m_2 \left[\frac{(1-\kappa)^2}{\lambda_2(1-\kappa) + \lambda_1 \kappa} + \frac{\kappa^2}{\lambda_1(1-\kappa) + \lambda_2 \kappa} \right] - (1 - 2\kappa + 2\kappa^2),$$

we obtain

$$\lambda_2^{max} = m_1 \left\{ \frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1 - 2\kappa + 2\kappa^2)} \right\},$$

and

$$\lambda_1^{max} = m_2 \left\{ \frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1 - 2\kappa + 2\kappa^2)} \right\}.$$

Observe that $\dfrac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1 - 2\kappa + 2\kappa^2)}$ is decreasing in κ, takes on the value 2 when $\kappa = 1/2$ and tends to infinity as κ goes to zero. This implies that $\lambda_1^{max} \geq 2m_2$ and $\lambda_2^{max} \geq 2m_1$.

Next, let $\lambda_1^{\times}(\lambda_2^{\times})$ denote the highest type representative that region 1's (region 2's) median voter would want. These levels are implicitly defined by the equalities

$$\partial U_1(\lambda_1^{\times}, 0)/\partial \lambda_1 = 0$$

and

$$\partial U_2(0, \lambda_2^{\times})/\partial \lambda_2 = 0,$$

which imply

$$\lambda_1^{\times} = \frac{m_1}{1 - 2\kappa + 2\kappa^2}$$

and

$$\lambda_2^{\times} = \frac{m_2}{1 - 2\kappa + 2\kappa^2}.$$

Note that $1/(1-2\kappa+2\kappa^2)$ is increasing in κ, and takes on the value 1 when $\kappa = 0$ and value 2 when $\kappa = 1/2$. This implies that $\lambda_1^{\times} \leq 2m_1$ and $\lambda_2^{\times} \leq 2m_2$. By assumption, $2m_i < \lambda^{max}$, so that the upper bound constraint on type choice is not binding here.

We may conclude from the above that for all $\lambda_2 \in \langle 0, \min\{\lambda_2^{max}, \lambda^{max}\}\rangle$, $r_1(\lambda_2)$ is implicitly defined by the first-order condition

$$\frac{\partial U_1(r_1(\lambda_2), \lambda_2)}{\partial \lambda_1} = 0$$

and for all $\lambda_2 \in (\min\{\lambda_2^{max}, \lambda^{max}\}, \lambda^{max}\rangle$,

$$r_1(\lambda_2) = 0.$$

We also know that $r_1(0) = \lambda_1^{\times}$ and that $r_1(\lambda_2)$ falls on $\langle 0, \min\{\lambda_2^{max}, \lambda^{max}\}\rangle$.

Analogously, for all $\lambda_1 \in \langle 0, \min\{\lambda_1^{max}, \lambda^{max}\}\rangle$, $r_2(\lambda_1)$ is implicitly defined by the first-order condition

$$\frac{\partial U_2(\lambda_1, r_2(\lambda_1))}{\partial \lambda_2} = 0$$

and for all $\lambda_1 \in (\min\{\lambda_1^{max}, \lambda^{max}\}, \lambda^{max}\rangle$,

$$r_2(\lambda_1) = 0.$$

Also, note that $r_2(0) = \lambda_2^{\times}$ and that $r_2(\lambda_1)$ declines in $\langle 0, \min\{\lambda_1^{max}, \lambda^{max}\}\rangle$.

We can now prove the lemma. If $\kappa < \hat{\kappa}$, it follows from the definition of $\hat{\kappa}$ in the text that $(\kappa^3 + (1-\kappa)^3)/\kappa(1-\kappa) > m_1/m_2$. This in turn implies that

$$\lambda_1^{max} = m_2\left\{\frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)}\right\} > \frac{m_1}{(1-2\kappa+2\kappa^2)} = \lambda_1^{\times}.$$

Observe further that

$$\lambda_2^{max} = m_1\left\{\frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1-2\kappa+2\kappa^2)}\right\} > 2m_2 \geq \lambda_2^{\times}.$$

These inequalities imply that there exists no boundary equilibria in which $\lambda_i^* = 0$ for one or more districts. If $\lambda_2^* = 0$, then $\lambda_1^* = r_1(0) = \lambda_1^{\times}$, but since $\lambda_1^{\times} < \lambda_1^{max}$ we know that $r_2(\lambda_1^{\times}) > 0$, which contradicts the fact that $\lambda_2^* = 0$. If $\lambda_1^* = 0$, then $\lambda_2^* = r_2(0) = \lambda_2^{\times}$, but since $\lambda_2^{\times} < \lambda_2^{max}$ we know that $r_1(\lambda_2^{\times}) > 0$, which contradicts the fact that $\lambda_1^* = 0$. Since $\max r_i(\lambda_{-i}) < \lambda^{max}$, it is apparent that there can be no boundary equilibria in which $\lambda_i^* = \lambda^{max}$ for one or more districts.

It follows that there must exist an interior equilibrium. Any such equilibrium $(\lambda_1^*, \lambda_2^*)$ must satisfy the first-order conditions $\partial U_i(\lambda_1^*, \lambda_2^*)/\partial \lambda_i = 0$ for $i \in \{1, 2\}$. Using the expressions for $\partial U_i/\partial \lambda_i$, $i \in \{1, 2\}$ from above, we may write these conditions as

$$m_1\left[\frac{(1-\kappa)^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} + \frac{\kappa^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa}\right] = (1-2\kappa+2\kappa^2),$$

and

$$m_2 \left[\frac{(1-\kappa)^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} + \frac{\kappa^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} \right] = (1 - 2\kappa + 2\kappa^2).$$

Combining the two first-order conditions, we obtain

$$\frac{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} = \frac{m_1(1-\kappa)^2 - m_2\kappa^2}{m_2(1-\kappa)^2 - m_1\kappa^2}.$$

Using this and the two first-order conditions for λ_1^* and λ_2^* yields

$$\lambda_1^*(1-\kappa) + \lambda_2^*\kappa = \frac{m_1 m_2[(1-\kappa)^4 - \kappa^4]}{[m_2(1-\kappa)^2 - m_1\kappa^2](1 - 2\kappa + 2\kappa^2)} = \frac{m_1[(1-\kappa)^2 - \kappa^2]}{(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2}$$

and

$$\lambda_2^*(1-\kappa) + \lambda_1^*\kappa = \frac{m_1 m_2[(1-\kappa)^4 - \kappa^4]}{[m_1(1-\kappa)^2 - m_2\kappa^2](1 - 2\kappa + 2\kappa^2)} = \frac{m_1[(1-\kappa)^2 - \kappa^2]}{\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2}.$$

Thus, as claimed, the policy outcome is

$$(g_1, g_2) = \left(\frac{m_1[(1-\kappa)^2 - k^2]}{\left[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2 \right] p}, \frac{m_1[(1-\kappa)^2 - k^2]}{\left[\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2 \right] p} \right).$$

If $\kappa \geq \hat{\kappa}$, it follows that $(\kappa^3 + (1-\kappa)^3)/\kappa(1-\kappa) \leq m_1/m_2$, which in turn implies that

$$\lambda_1^{max} = m_2 \left\{ \frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)(1 - 2\kappa + 2\kappa^2)} \right\} \leq \frac{m_1}{(1 - 2\kappa + 2\kappa^2)} = \lambda_1^\times.$$

This inequality implies that there exists a boundary equilibrium in which $(\lambda_1^*, \lambda_2^*) = (\lambda_1^\times, 0)$. This is because $r_2(\lambda_1^\times) = 0$ and $r_1(0) = \lambda_1^\times$. the same arguments from above imply that there exist no other boundary equilibria. We also claim that there are no interior equilibria. Any such equilibrium $(\lambda_1^*, \lambda_2^*)$ must satisfy the first-order conditions $\partial U_i(\lambda_1^*, \lambda_2^*)/\partial \lambda_i = 0$ for $i \in \{1, 2\}$. These first-order conditions imply that

$$m_1[\lambda_2^*(1-\kappa)^3 + \lambda_1^*\kappa(1-\kappa)^2 + \lambda_1^*(1-\kappa)\kappa^2 + \lambda_2^*\kappa^3] =$$
$$= m_2[\lambda_1^*(1-\kappa)^3 + \lambda_2^*\kappa(1-\kappa)^2 + \lambda_2^*(1-\kappa)\kappa^2 + \lambda_1^*\kappa^3].$$

This means that

$$\lambda_2^* = \frac{[m_2((1-\kappa)^3 + \kappa^3) - m_1\kappa(1-\kappa)]}{[m_1((1-\kappa)^3 + \kappa^3) - m_2\kappa(1-\kappa)]} \lambda_1^*.$$

But the assumption that $\kappa \geq \hat{\kappa}$ implies that $\lambda_2^* \leq 0$ if $\lambda_1^* > 0$, which, in turn, is inconsistent with the hypothesis that $(\lambda_1^*, \lambda_2^*) > (0, 0)$. Thus, the only equilibrium is that

$(\lambda_1^*, \lambda_2^*) = (\lambda_1^\times, 0) = \left(\dfrac{m_1}{1 - 2\kappa + 2\kappa^2}, 0 \right) = \left(\dfrac{m_1}{(1-\kappa)^2 + \kappa^2}, 0 \right)$. The required policy outcomes follow from this equilibrium:

$$(g_1, g_2) = \left(\frac{m_1(1-\kappa)}{[(1-\kappa)^2 + \kappa^2]p}, \frac{m_1 \kappa}{[(1-\kappa)^2 + \kappa^2]p} \right). \qquad \square$$

Proof of Proposition 4. When the regions are identical $(m_1 = m_2 = m)$, the surplus under centralization with cooperative legislature and

(a) uniform taxation is $S_u^c(\kappa) = 2m \ln \dfrac{2m(1 - 2\kappa + 2\kappa^2)}{p} - 4m(1 - 2\kappa + 2\kappa^2)$;

(b) non-uniform taxation is $S_n^c(\kappa) = 2m \ln \dfrac{m}{p} - 2m$.

We establish six claims from which the proposition will follow.

CLAIM 1. $S_u^c(0) < S^d(0)$ and $S_u^c(1/2) > S^d(1/2)$.
Calculating the first inequality, we obtain

$$S_u^c(0) = 2m \ln \frac{2m}{p} - 4m < 2m \ln \frac{m}{p} - 2m = S^d(0)$$

$$\ln \frac{2m}{p} - 2 < \ln \frac{m}{p} - 1$$

$$\ln 2 < 1,$$

which holds. Computing the second inequality, we come to $\ln 2 > 1/2$, which is also true.

CLAIM 2. $S_u^c(\kappa)$ is increasing in spillovers.
Differentiating $S_u^c(\kappa)$, we obtain

$$\frac{\partial S_u^c}{\partial \kappa}(\kappa) = 4m(1 - 2\kappa) \left[\frac{(1 - 2\kappa)^2}{(1-\kappa)^2 + \kappa^2} \right] > 0 \quad \text{for all } \kappa \in (0, 1/2).$$

CLAIM 3. $S^d(\kappa)$ is decreasing in κ.
This claim has already been proven in the Proof of Proposition 1.

CLAIM 4. $S_n^c(0) = S^d(0)$.

CLAIM 5. $S_n^c(\kappa)$ is constant in spillovers.
Both claims are straightforward.

CLAIM 6. $S_u^c(1/2) = S_n^c(1/2)$.
This can be easily proven by inserting $1/2$ into the functions of the surpluses.

Part (i) of the proposition follows from Claims 1, 2 and 3. Claims 3, 4 and 5 imply part (ii) of the proposition. Finally, Claim 6, combined with Claims 2 and 5, implies part (iii) of the proposition. $\square$

Proof of Proposition 5. Considering non-identical districts $(m_1 > m_2)$, denote the policy outcomes under centralization with a cooperative legislature for uniform taxation as $(g_1^{c-u}(\kappa), g_2^{c-u}(\kappa))$ and non-uniform taxation as $(g_1^{c-n}(\kappa), g_2^{c-n}(\kappa))$. Then the surplus

(a) with uniform taxation is

$$S_u^c(\kappa) = [m_1(1-\kappa) + m_2\kappa] \ln g_1^{c-u}(\kappa) + [m_2(1-\kappa) + m_1\kappa] \ln g_2^{c-u}(\kappa)$$
$$- p(g_1^{c-u}(\kappa) + g_2^{c-u}(\kappa)),$$

(b) and with non-uniform taxation is

$$S_n^c(\kappa) = [m_1(1-\kappa) + m_2\kappa] \ln g_1^{c-n}(\kappa) + [m_2(1-\kappa) + m_1\kappa] \ln g_2^{c-n}(\kappa)$$
$$- p(g_1^{c-n}(\kappa) + g_2^{c-n}(\kappa)).$$

We prove the proposition via five claims.

CLAIM 1. $S_u^c(0) < S^d(0)$.
Computing this inequality we come to the following one: $\ln 2 < 1$, which clearly holds.

CLAIM 2. $S_u^c(1/2) > S^d(1/2)$.
Let (m_1, m_2) be given. We can find $\omega > 0$ and $\alpha \in (1/2, 1)$ such that $(m_1, m_2) = (\alpha\omega, (1-\alpha)\omega)$. In addition, since $\hat{\kappa} < 1/2$, we have that

$$g_1^{c-u}\left(\tfrac{1}{2}\right) = g_2^{c-u}\left(\tfrac{1}{2}\right) = \frac{\alpha\omega}{p}.$$

It follows that

$$S_u^c\left(\tfrac{1}{2}, \alpha\right) = \omega \ln \frac{\alpha\omega}{p} - 2\alpha\omega.$$

Under decentralization, the surplus is given by

$$S^d\left(\tfrac{1}{2}, \alpha\right) = \frac{\omega}{2}\left[\ln \frac{\alpha\omega}{2p} + \ln \frac{(1-\alpha)\omega}{2p}\right] - \frac{\omega}{2}.$$

Calculating the difference, we obtain

$$S_u^c\left(\tfrac{1}{2}, \alpha\right) - S^d\left(\tfrac{1}{2}, \alpha\right) = \omega \ln \frac{\alpha\omega}{p} - 2\alpha\omega - \frac{\omega}{2}\left[\ln \frac{\alpha\omega}{2p} + \ln \frac{(1-\alpha)\omega}{2p}\right] + \frac{\omega}{2}$$
$$= \frac{\omega}{2}\left[\ln \frac{\alpha}{1-\alpha}\right] - 2\alpha\omega + \omega \ln 2 + \frac{\omega}{2}.$$

Differentiating the difference with respect to α, we obtain

$$\frac{\partial\left[S_u^c(1/2, \alpha) - S^d(1/2, \alpha)\right]}{\partial\alpha} = \omega \frac{(1-2\alpha)^2}{2\alpha(1-\alpha)} \geq 0.$$

Thus, this difference is non-decreasing in α. So if $S_u^c(1/2, 1/2) > S^d(1/2, 1/2)$, then the inequality holds for all α in the relevant range. But $\alpha = 1/2$ corresponds to the identical districts case and we already know that the surplus under centralization is higher than that under decentralization then.

CLAIM 3. $S_n^c(0) = S^d(0)$.
This is easily verified by inserting 0 into the corresponding functions of the surpluses.

CLAIM 4. $\dfrac{\partial \left[S_n^c(0) - S^d(0)\right]}{\partial \kappa} > 0$ for all $\kappa \in (0, \varepsilon)$, where $\varepsilon > 0$.
This claim holds, but due to its algebraic difficulty we will not perform the proof here.

CLAIM 5. $S_n^c\left(\frac{1}{2}, \alpha\right) = S_u^c\left(\frac{1}{2}, \alpha\right)$.
This statement is easily checked. We leave this to the reader.

For the first half of part (i) of Proposition 5, we employ Claim 1 and the fact that both surplus functions are continuous functions of κ. Then for each (m_1, m_2) there exists $\delta > 0$ such that $S_u^c(\kappa) < S^d(\kappa)$ for all $\kappa < \delta$. A similar logic with Claim 2 establishes the second half of (i).

For part (ii), by utilizing Claims 5 and 2 we can prove that $S_n^c(1/2) > S^d(1/2)$. Since the surplus function for the non-uniform taxation case is a continuous function of κ, for each (m_1, m_2) we can find $\delta > 0$ such that $S_n^c(1/2 - \kappa) > S^d(1/2 - \kappa)$ for all $\kappa < \delta$. the latter half of (ii) is proven by employing Claims 3 and 4 and using the fact that both surplus functions are continuous functions of κ.

Part (iii) is algebraically tedious and is available upon request. □