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NEW MEASURE OF VOTING POWER

Abstract:
During last two decades we observe a boom of power indices literature related to constitutional analysis of European Union institutions and distribution of intra-institutional and inter-institutional influence in the European Union decision making. Growing interest to power indices methodology leads also to reconsideration of the methodology itself. In this paper a new general a priori voting power measure is proposed distinguishing between absolute and relative power. This power measure covers traditional Shapley-Shubik and Penrose-Banzhaf power indices as its special cases.

Keywords: absolute power, cooperative games, decisive situation, *I*-power, pivot, power indices, *P*-power, relative power, swing

JEL classification: D710, D740

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1. Introduction

Power indices methodology is widely used to measure a priori voting power of members of a committee. Two most widely used power indices were proposed by Penrose and Banzhaf (1946, 1965) and Shapley and Shubik (1954). We shall refer to them as PB-power index and SS-power index. There exist also some other well-defined power indices, such as Holler-Packel index (1983), Johnston index (1978), and Deegan-Packel index (1979). The most comprehensive survey and analysis of power indices methodology see in (Felsenthal, Machover, 1998).

During last two decades we observe a boom of power indices literature related to constitutional analysis of European Union institutions and distribution of intra-institutional and inter--institutional influence in the European Union decision making. Distribution of power in the EU Council of Ministers and European Parliament has been analyzed earlier in (Holler, Kellermann, 1977), (Johnston, 1982), (Brams, Affuso, 1985), the recent development associated with the 1995 enlargement of the EU in (Widgrén, 1993, 1994, 1995), (Berg, Lane, 1997), (Lane, Maeland, 1996), (Nurmi, 1997a), (Nurmi, Meskanen, Pajala, 2001), (Bindseil, Hantke, 1997), (Felsenthal, Machover, 1998), (Mercik, 1999), (Turnovec, 1996, 2001, 2002, 2003), (Kauppi, Widgrén, 2004) and others. Holler and Widgrén (1999) provide strong arguments for power indices methodology in assessing EU decision-making. What exactly power indices are measuring is controversial, see e.g. arguments of Garrett and Tsebelis (1999) about ignoring preferences, and

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response of Holler and Widgrén (1999), but they are of general interest to political science because they may measure players ability to get what they want. Admittedly significant share of decisions under the EU decision making procedures are taken without recourse to a formal vote. But it may well be the case that the outcome of negotiation is conditioned by the possibility that a vote could be taken, and than a priori evaluation of voting power matters. Moreover, analyses of institutional design of decision making could benefit from power indices methodology (Holler, Owen 2001), (Lane, Berg, 1999). Continuing research and deeper understanding of power indices methodology reflect an actual demand for amendment of traditional legal and political analysis of institutional problems by quantitative approaches and arguments.

In this paper we analyse Shapley-Shubik and Penrose-Banzhaf concepts of power measure and classification of so called *I*-power (voter's potential influence over the outcome of voting) and *P*-power (expected relative share in a fixed prize available to the winning group of committee members) introduced by Felsenthal, Machover and Zwicker (1998). We show that objections against Shapley-Shubik power index, based on its interpretation as a *P*-power concept, are not sufficiently justified. Both Shapley-Shubik and Penrose-Banzhaf measure could be successfully derived as cooperative game values, and at the same time both of them can be interpreted as probabilities of being in some decisive position (pivot, swing) without using cooperative game theory at all, see also (Turnovec, 2004) and (Turnovec, Mercik, Mazurkiewicz, 2004).

It is demonstrated in the paper that both pivots and swings can be introduced as special cases of a more general concept of decisiveness based on assumption of equi-probable orderings expressing intensity of committee members' support for voted issues. New general a priori voting power measure is proposed distinguishing between absolute and relative power. This power measure covers Shapley-Shubik and Penrose-Banzhaf indices as its special cases.

2. Basic Concepts

Let $N = \{1, ..., n\}$ be the set of members (players, parties) and ω_i ($i = 1, ..., n$) be the (real, non-negative) weight of the i-th member such that

$$\sum_{i \in N} \omega_i = \tau, \omega_i \geq 0$$

(e.g. the number of votes of party i, or the ownership of i as a proportion of the total number of shares), where τ is the total weight of all members. Let γ be a real number such that $0<\gamma<\tau$. The $(n + 1)$-tuple

$$[\gamma, \omega] = [\gamma, \omega_1, \omega_2, ..., \omega_n]$$

such that

$$\sum_{i=1}^{n} \omega_i = \tau, \omega_i \geq 0, 0 \leq \gamma \leq \tau$$

we shall call a committee (or a weighted voting body) of the size n with quota γ, total weight τ and the allocation of weights

$$\omega = (\omega_1, \omega_2, ..., \omega_n)$$

Any non-empty subset $S \subseteq N$ we shall call a voting configuration. Given an allocation ω and a quota γ, we shall say that $S \subseteq N$ is a winning voting configuration, if

$$\sum_{i \in S} \omega_i \geq \gamma$$

and a losing voting configuration, if

$$\sum_{i \in S} \omega_i < \gamma$$

A configuration S is winning, if it has a required majority, otherwise it is losing.

Let

$$G = \left[(\gamma, \omega) \in R_{n+1} : \sum_{i=1}^{n} \omega_i = \tau,\ \omega_i \geq 0,\ 0 \leq \gamma \leq \tau \right]$$

be the space of all committees of the size n, total weight τ and quota γ.

A *power index* is a vector valued function

$$\Pi : G \rightarrow R_n$$

that maps the space G of all committees into R_n. A power index represents for each of the committee members a reasonable expectation that she will be "decisive" in the sense that her vote (YES or NO) will determine the final outcome of voting. The probability to be decisive we call an absolute power index of an individual member, by normalization of absolute power index we obtain a relative power of an individual member. By $\Pi_i(\gamma, \omega)$ and $\pi_i(\gamma, \omega)$ we denote the absolute and relative power the index grants to the i-th member of a committee with weight allocation ω, total weight τ and quota γ.

To define a particular power measure means to identify some qualitative property (decisiveness) whose presence or absence in voting process can be established and quantified (Nurmi, 1997a). Generally there are two such properties, related to committee members' positions in voting, that are being used as a starting point for quantification of an a priori voting power: swing position and pivotal position of committee members.

3. Penrose-Banzhaf and Shapley-Shubik: Swings and Pivots

Two most widely used power indices were proposed by Penrose and Banzhaf (1946, 1965) and Shapley and Shubik (1954). We shall refer to them as PB-power index and SS-power index.

The PB-power measure is based on the concept of swing. Let S be a winning configuration in a committee [γ, $\boldsymbol{\omega}$] and i $\in$ S. We say that a member i has a swing in configuration S if

$$\sum_{k \varepsilon S} \omega_k \geq \gamma \quad \text{and} \quad \sum_{k \varepsilon S \setminus \{i\}} \omega_k < \gamma$$

Let s_i denotes the total number of swings of the member i in the committee [γ, ω]. Then PB-power index is defined as

$$\pi_i^{PB}(\gamma, \omega) = \frac{s_i}{\sum_{k \in N} s_k}$$

In literature this form is usually called a relative PB-index. Original Penrose definition of power of the member i was

$$\pi_i^{P}(\gamma, \omega) = \frac{s_i}{2^{n-1}}$$

which, assuming that all configurations are equally likely, is nothing else but the probability that the given member will be decisive (the probability to have a swing). In literature this form is usually called an absolute PB-index. The relative PB-index is obtained by normalization of the absolute PB--index.

Also some other frequently quoted power indices belong to the class of swing-based measures of power: Holler-Packel index, Johnston index, and Deegan-Packel index.

Let the numbers 1, 2, ..., n be fixed names of committee members. Let

$$(i_1, i_2, ..., i_n)$$

be a permutation of the members of the committee, and let us assume that member k is in position r in this permutation, i.e. $k = i_r$. We shall say that the member k of the committee is in a pivotal situation (has a pivot) with respect to a permutation (i_1, i_2, ..., i_n), if

$$\sum_{j=1}^{r-1} \omega_{i_j} < q \wedge \sum_{j=1}^{r} \omega_{i_j} \geq q$$

The SS-power index is based on the concept of pivot. Let us assume that a strict ordering of members in a given permutation expresses an intensity of their support (preferences) for a particular issue in the sense that, if a member i_s precedes in this permutation a member i_t, then support by i_s for the particular proposal to be decided is stronger than support by i_t. One can assume that the group supporting the proposal will be formed in the order of positions of members in the given permutation. If it is so, then the member k will be in situation when the group composed from preceding members in the given permutation still does not have enough of votes to pass the proposal, and a group of members place behind him in the permutation has not enough of votes to block the proposal. The group that will manage his support will win. Members in a pivotal situation have a decisive influence on the final outcome. Assuming many voting acts and all possible preference orderings equally likely, under the full veil of ignorance about other aspects of individual members preferences, it makes sense to evaluate an a priori voting power of each committee member as a probability of being in pivotal situation. This probability is measured by the SS-power index:

$$\pi_i^{SS}(\gamma, \omega) = \frac{p_i}{n!}$$

where p_i is the number of pivotal positions of the committee member i and n! is the number of permutations of the committee members, i.e., number of different orderings of n elements.

4. Committee Models and Cooperative Games

Felsenthal et al. (1998) introduced concept of so called *I*-power and *P*-power. By *I*-power Felsenthal and Machover (2003, p.8) mean "voting power conceived of as a voter's potential influence over the outcome of divisions of the decision making body: whether proposed bills are adopted or blocked. Penrose's approach was clearly based on this notion, and his measure of voting power is a proposed formalization of a priori *I*-power." By *P*-power they mean "voting power conceived as a voter's expected relative share in a fixed prize available to the winning coalition under a decision rule, seen in the guise of a simple TU (transferable utility) cooperative game. The Shapley-Shubik approach was evidently based on this notion, and their index is a proposed quantification of a priori *P*-power." Hence, the fundamental distinction between *I*-power and *P*-power is in the fact that the *I*-power notion takes the outcome to be the immediate one, passage or defeat of the proposed bill, while the *P*-power view is that passage of the bill is merely the ostensible and proximate outcome of a division; the real and ultimate outcome is the distribution of fixed a purse – the prize of power – among the winners (Felsenthal, Machover, 2003, pp. 9–10). As a conclusion it follows that *SS*-power index does not measure a priori voting power, but says how to agree on dividing the "pie" (benefits of victory).

As the major argument of this classification the authors provide a historical observation: Penrose paper of 1946 was ignored and unnoticed by mainstream – predominantly American – social choice theorists, and Shapley and Shubik's 1954 paper was seen as inaugurating the scientific study of voting power. Because the Shapley-Shubik paper was wholly based on cooperative game theory, it induced among social scientists an almost universal unquestioning belief that the study of

power was necessarily and entirely a branch of that theory (Felsenthal, Machover, 2003, p. 8). Conclusion follows, that since the cooperative game theory with transferable utility is about how to divide a pie, and *SS*-power index was derived as a special case of Shapley value of cooperative game, the *SS*-power index is about *P*-power and does not measure voting power as such.

We demonstrated above that one does not need cooperative game theory to define and justify *SS*-power index. *SS*-power index is a probability to be in a pivotal situation in an intuitively plausible process of forming a winning configurations, no division of benefits is involved. Incidentally *SS*-power index appeared as an interesting special case of Shapley value for cooperative games with the transferable utility, but in exactly the same way one can handle the PB-index. Let us make a short excursion into cooperative game theory.

Let N be the set of players in a cooperative game (cooperation among the players is permitted and the players can form coalitions and transfer utility gained together among themselves) and 2^N its power set, i.e. the set of all subsets $S \subseteq N$, called coalitions, including empty coalition. Characteristic function of the game is a mapping

$$v : 2^N \to R$$

with

$$v(\varnothing) = 0$$

($\varnothing$ stands for empty set.) The interpretation of v is that for any subset S of N the number $v(S)$ is the value (worth) of the coa-lition S, in terms how much "utility" the members of S can divide among themselves in any way that sums to no more than $v(S)$ if they all agree. The characteristic function is said to be super-addi-tive if for any two disjoint subsets $S, T \subseteq N$ we have

$$v(S \cup T) \geq v(S) + v(T)$$

i.e. the worth of the coalition $S \cup T$ is equal to at least the worth of its parts acting separately. Let us denote cooperative characteristic function form by $[N, v]$. The game $[N, v]$ is said to be super--additive if its characteristic function is super-additive.

By a value of the game $[N, v]$ we mean a non-negative vector $\varphi(v)$ such that

$$\sum_{i \in N} \varphi_i(v) = v(N)$$

By

$$c(i, T) = v(T) - v(T - \{i\})$$

we shall denote marginal contribution of the player $i \in N$ to the coalition $T \subseteq N$. Then, in an abstract setting, the value $\varphi_i(v)$ of the i-th player in the game $[N, v]$ can be defined as a weighted sum of his marginal contributions to all possible coalitions he is member of:

$$\varphi_i(v) = \sum_{T \subseteq N, i \in T} \alpha(T) c(i, T)$$

Different weights $\alpha(T)$ leads to different definitions of values. Shapley (1953) defined his value by the weights

$$\alpha(T) = \frac{(t-1)!(n-t)!}{n!}$$

where t = card (T). He proved that it is the only value that satisfies the following three axioms: (i) dummy axiom (dummy player, i.e. the player that contributes nothing to any coalition, has zero value), (ii) permutation axiom (for any game $[N, u]$ that is generated from the game $[N, v]$ by a permutation of players, the value $\varphi(u)$ is a corresponding permutation of the value $\varphi(v)$) and (iii) additivity axiom (for sum $[N, v+u]$ of two games $[N, v]$ and $[N, u]$ the value $\varphi(v + u) = \varphi(v) + \varphi(u)$).

As Owen (1982) noticed, the relative PB-index is meaningful for general cooperative games with transferable utilities. One can define Banzhaf value by setting the weights

$$\alpha(T) = \frac{v(N)}{\sum_{k \in N, T \subseteq N} c(k,T)}$$

Owen (1982) shows a certain relation between the Shapley value and Banzhaf value of cooperative game with transferable utilities: both give averages of player's marginal contributions, the difference lies in the weighting coefficients (in the Shapley value coefficients depends on size of coalitions, in the Banzhaf value they are independent of coalition size).

A similar generalization of the Holler-Packel public good index (which is based on membership in minimal winning configurations, in which each member has a swing) as a cooperative game value see in (Holler, Xiaoguang Li, 1995).

The relation between values and power indices is straightforward: A cooperative characteristic function game represented by a characteristic function v such that v takes only the values 0 and 1 is called a simple game. With any committee with quota γ and allocation w we can associate a super-additive simple game such that

$$v(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} \omega_i \geq \gamma \\ 0 & \text{otherwise} \end{cases}$$

i.e. a coalition has value 1 if it is winning and value 0 if it is losing.

Super-additive simple games can be used as natural models of voting in committees. Shapley and Shubik (1954) applied the concept of the Shapley value for general cooperative characteristic function games to the super-additive simple games as a measure of voting power in committees. Here we demonstrated that PB relative power index can be extended as a value for general cooperative characteristic function games (for an axiomatization, see (Owen, 1982). But we do not need cooperative games to model voting in committees.

5. Pivots and Swings: Special Cases of a More General Concept of Decisiveness

Let us consider a binary relation $r\ R\ t$ ($r, t \in N$) meaning "member r's support for some issue is not weaker than member t's support". Let $(S_1, S_2, \ldots, S_k)$, $k \leq n$, be a partition of N, i.e.,

$$\bigcup_{j=1}^{k} S_j = N$$

and for any $s, t \in N$, $s \neq t$, it holds that $S_s \cap S_t = \varnothing$. Let $(j_1, j_2, \ldots, j_k)$ be a permutation of numbers $(1, 2, \ldots, k)$, then $S = (S_{j1}, S_{j2}, \ldots, S_{jk})$ we shall call an ordering defined on N. Considering a particular issue, there is the following interpretation of the ordering on N: If $r \in S_{ju}$ and $t \in S_{jv}$ and $u<v$, then a member r's support for the particular proposal is stronger than the support of member t. If $r, t \in S_{ji}$, then the intensity of support for a particular proposal by the both r and t is the same. Then, if $u<v$, it is plausible to assume, that if members from S_{iv} vote YES, then the members from S_{iu} also vote YES, if the members from S_{iu} vote NO, then the members from S_{iv} also vote NO.

Let k ($1 \leq k \leq n$) be the number of sets S_{ji} in S. If $k = n$, then S is a strict ordering (no indifferences), if $k = 2$, then S is a binary ordering (only YES and NO groups). Let us denote by $P(N)$ set of all partitions on N and by $W(N)$, $S(N)$ and $B(N)$ set of all orderings, strict orderings and binary orderings on N. Using Stirling's formula we can prove that

$$\mathrm{card}(W(N)) = \sum_{k=1}^{n} \frac{1}{k!} \sum_{j=0}^{k-1} (-1)^j \binom{k}{j} \ (k-j)^n$$

(number of all orderings on N),

$$\mathrm{card}(P(N)) = \sum_{k=1}^{n} \sum_{j=0}^{k-1} (-1)^j \binom{k}{j} \ (k-j)^n$$

(number of partitions on N). Particularly, card $(S(N)) = n!$, and card $(B(N)) = 2^n-1$.

From formal reason we shall denote $S_{i0} = \{0\}$ and $\omega_0 = 0$. Let $S = (S_1, \ldots, S_v, \ldots, S_k)$ be an ordering. We shall say that a group S_v is in a pivotal position in ordering S, if

$$\sum_{j=0}^{v-1} \sum_{i \in S_j} \omega_i < \gamma \quad \text{and} \quad \sum_{j=1}^{v} \sum_{i \in S_j} \omega_i \geq \gamma$$

Under our assumptions, if all members of the pivotal group vote YES, then the outcome of voting is YES, if they vote NO, then the outcome is NO.

We shall say that a member $t \in S_{iv}$ (v>0) is in a decisive situation in an ordering $(S_{i1}, S_{i2}, \ldots, S_{ik})$ if S_{iv} is pivotal group and $S_{iv} \backslash \{t\}$ is not a pivotal group.

Let the group S_v is pivotal and $t \in S_v$, then t is decisive if and only if either

$$\sum_{j=0}^{v-1} \sum_{i \in S_j} \omega_i + \sum_{i \in Si_v \backslash \{t\}} \omega_i < \gamma$$

(i.e., if the group S_v joins preceding groups voting YES, then by changing unilaterally his YES to NO member t changes the outcome of voting from YES to NO), or

$$\sum_{j=0}^{v-1} \sum_{i \in S_j} \omega_i + \omega_t \geq \gamma$$

(i.e., if the group S_v does not join preceding groups voting YES, then by changing unilaterally his NO to YES member t changes the outcome of voting from NO to YES).

Applying this definition to $S \in S(N)$, we obtain concept of pivot, applying it to $S \in B(N)$, we obtain concept of swing.

Then it makes sense to measure a priori voting power of the member of the committee by number of his/her decisive situations. Assuming that in sufficiently large number of voting acts all orderings (expressing preferences of members) are equally possible (equi-probable), then measure of so called absolute power of the member i is given by the ratio of the number of decisive situations of the member i to the number of orderings, and relative power is given by the ratio of the number of decisive situations of the member i to the total number of decisive situations

Assume that $D(N) \subseteq W(N)$ is a set of equi-probable orderings one considers relevant for a priori voting power evaluation. Let us define

$$\rho_i(S) = \begin{Bmatrix} 1 & \text{if } i \text{ is decisive in } (S) \\ 0 & \text{otherwise} \end{Bmatrix}$$

where $S \in D(N)$. Absolute power of the member i with respect to $D(N)$ decisiveness is defined as

$$\Pi_i = \frac{\sum_{S \in D(N)} \rho_i(S)}{\mathrm{card}(D(N))}$$

(probability that i will be in decisive position), while the relative power of the member i is defined as

$$\pi_i = \frac{\sum_{S \in D(N)} \rho_i(S)}{\sum_{k \in N} \sum_{S \in D(N)} \rho_k(S)}$$

(share of one voter's power).

Selecting $D(N) = S(N)$, we obtain Shapley-Shubik power index, selecting $D(N) = B(N)$ we obtain Penrose-Banzhaf index. If we select $D(N) = W(N)$, we obtain a more general measure of a priori voting power, based on plausible assumption about possible individual attitudes to the voted issues.

6. Numerical Example

To illustrate the concepts introduced let us use a simple example. Let $N = \{1, 2, 3\}$. Consider a committee [51; 50, 30, 20]. In this case we have 13 orderings: (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1), (1, (2, 3)), (2, (1,3)), (3, (1,2)), ((1, 2), 3), ((2, 3), 1), ((1, 3), 2), ((1, 2, 3)), including 6 strict orderings (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1) and 7 binary orderings (1, (2, 3)), (2, (1,3)), (3, (1,2)), ((1, 2), 3), ((2, 3), 1), ((1, 3), 2), ((1, 2, 3)). The following table provides list of orderings and decisive situations:

ordering	σ_1	σ_2	σ_3	Σ		
(1, 2*, 3)	0	1	0	1		SS
(1, 3*, 2)	0	0	1	1		SS
(2, 1*, 3)	1	0	0	1		SS
(2, 3, 1*)	1	0	0	1		SS
(3, 1*, 2)	1	0	0	1		SS
(3, 2, 1*)	1	0	0	1		SS
(1, (2*, 3*))	0	1	1	2		PB
(2, (1*,3))	1	0	0	1		PB
(3, (1*,2))	1	0	0	1		PB
((1*, 2*), 3)	1	1	0	2		PB
((2, 3), 1*)	1	0	0	1		PB
((1*, 3), 2)	1	0	1	2		PB
((1*,2,3))	1	0	0	1		PB
Σ	10	3	3	16	13	all orderings
Σ	4	1	1	6	6	strict orderings
Σ	6	2	2	10	7	binary orderings

By asterisk we denote decisive member, by SS the orderings relevant for Shapley-Shubik index, by PB the orderings relevant for Penrose-Banzhaf index.

General case (G power index): power measure based on assumption of equi-probable (all) orderings. Absolute power

$$\Pi_1 = 10/13,\ \Pi_2 = 3/13,\ \Pi_3 = 3/13$$

and relative power

$$\pi_1 = 10/16,\ \pi_2 = 3/16,\ \pi_3 = 3/16$$

Shapley-Shubik power index is based on strict orderings only. Absolute power

$$\Pi_1 = 4/6,\ \Pi_2 = 1/6,\ \Pi_3 = 1/6$$

and relative power

$$\pi_1 = 4/6,\ \pi_2 = 1/6,\ \pi_3 = 1/6$$

SS absolute power index is defined by the ratio of decisive situations of the member i (so called pivots) to the number of strict orderings, and SS relative power index is defined by the ratio of the decisive situations of i to the total number of strict orderings. Number of strict orderings is always equal to the number of decisive situations = $n!$, therefore there is no difference between absolute and relative power in Shapley-Shubik case.

Penrose-Banzhaf power index considers only binary orderings (partition on YES and NO voters). PB absolute power index is defined by the ratio of decisive situations of the member i in binary orderings (so called swings) to the number of binary orderings[1], and PB relative power index is defined by the ratio of the decisive situations of i to the total number of decisive situations in binary orderings. Absolute power

$$\Pi_1 = 6/7,\ \Pi_2 = 2/7,\ \Pi_3 = 2/7$$

and relative power

$$\pi_1 = 6/10,\ \pi_2 = 2/10,\ \pi_3 = 2/10$$

It is possible to prove that for the absolute G power index axioms of dummy member[2] (zero power of dummy member), symmetry[3] (equal power for symmetric members), anonymity (independence of power on permutation of members), local monotonicity (the member with greater weight cannot have less power than the member with smaller weight) and global monotonicity (if the weight of one member is increasing and the weights of all other members are decreasing or staying the same, then the power of the "growing weight" member will at least not decrease) are satisfied. More about axiomatic treatment see e.g. in (Allingham, 1975) and (Turnovec 1998). Some of these axioms might not be satisfied by relative indices (global monotonicity).

7. Concluding Remarks

There is no contradiction between Shapley-Shubik and Penrose-Banzhaf measure of voting power, no fundamental distinction between pivots and swings. Both concepts appear to be special cases of a more general concept based on full range of possible preferences of individual committee members and their groups, they follow from the same logic and could be formulated in the same framework.

Moreover, we do not need cooperative game theory to define and analyze a priori voting power, in some sense game-theoretical setting of the problem restricts analytical tools.

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[1] Note that by our definition of decisiveness we consider as decisive situations "negative" swings (ability to transform a winning configuration to the losing one) and "positive" swings (ability to transform a losing configuration to the winning one), while Penrose-Banzhaf measure works only with negative swings. The number of negative swings is equal to the number of positive swings, thus the Penrose-Banzhaf definition of absolute power index (number of i-th member negative swings)/($2n$–1) gives the same values as (total number of decisive situations)/($2n$). Considering only $2n$–1 binary orderings (not $2n$ bipartitions), we have by our definition (total number of decisive situations)/($2n$–1), which is proportional to Penrose-Banzhaf: if Π_i is traditional absolute PB-index and Π'_i is index based on binary orderings, then $\Pi'_i = (2n/2n-1)\Pi_i$.

[2] A member $i \in N$ of the committee $[\gamma, \omega]$ is said to be dummy if he cannot benefit any voting configuration by joining it, i.e. the player i is dummy if

$$\sum_{k \in S} \omega_k \geq \gamma \Rightarrow \sum_{k \in S - \{i\}} \omega_k \geq \gamma$$

[3] Two distinct members i and j of a committee $[\gamma, \omega]$ are called symmetric if their benefit to any voting configuration is the same, that is, for any S such that $i, j \notin S$

$$\sum_{k \in S \cup i} \omega_k \geq \gamma \ \Rightarrow \ \sum_{k \in S \cup j} \omega_k \geq \gamma$$

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