Czech Economic Review 9 (2015) 91–103
Acta Universitatis Carolinae Oeconomica

Insurance-Markets Equilibrium with Double Indivisible Labor Supply

Aleksandar Vasilev*

Received 4 December 2015; Accepted 15 December 2015

Abstract This note describes the lottery- and insurance-market equilibrium in an economy with both private and public sector employment and non-convex labor supply. In addition, when households are constrained to search for jobs only in a certain sector, the framework requires that there should be separate insurance markets: a public and a private sector one, which would pool the unemployment risk of the corresponding group of households. The unemployment insurance market segmentation is a new result in the literature and a direct consequence of the non-convexity of the labor supply in each sector and the sorting effect of the sector-type shock introduced in the model setup.

Keywords indivisible labor, public employment, insurance
JEL classification H31, J21

1. Introduction

Changes in hours account for approximately two-thirds of the cyclical output volatility in the standard real business cycle model (Cooley and Prescott 1995, Kydland 1995). Those hours, however, are assumed to be supplied in the private sector only, and thus the private-public sector labor choice is ignored. While this might be a reasonable assumption for the US economy, it comes in a stark contrast with the European Union (EU) evidence—after all, central governments in EU countries are the biggest employers at a national level, and public employment is a significant share of total employment.

This note adds to the literature by distinguishing between the two types of labor supply decisions by focusing on the fact that most of the volatility in hours is driven by volatility in employment. That is, most workers in Europe are employed full-time and in addition, only very rarely move between public and/or private sector, as documented in Gomes (2012).[1] Thus, the non-convex labor supply decisions (either work a full week on a job, or not work at all) in both sectors are taken under scrutiny, and the note will try to uncover whether this double binary labor supply decision could provide new implications for business cycle fluctuations in different EU member states.

* CERGE-EI Affiliate Fellow, Prague; Department of Economics, American University in Bulgaria, 1 Georgi Izmirliev Sq., Blagoevgrad 2700, Bulgaria. Tel.: 00 359 73 888 482, E-mail: avasilev@aubg.edu.

[1] In the setup, we model this lack of mobility between sectors via a shock process that sorts workers into a private-sector or a public-sector type.

In an earlier paper, Vasilev (2015a) extends Rogerson's (1988) and Hansen's (1985) static setup by augmenting it with a public sector, and introducing a shock that determines each household's type to be either private-sector or public-sector. The households then search for work in the sector corresponding to their type. Vasilev (2015a) then aggregates over individual households' utility functions, and finds that the resulting utility representation features constant, but different disutility of labor in the two sectors. The aggregate utility function then can not only accommodates the fact that average public sector wages feature a significant mark-up over private sector ones, as documented in Vasilev (2015b), but also allows for an additional transmission and propagation mechanism of shocks through the endogenous public sector labor choice.

In contrast to this earlier study, the focus of the present note falls on the lottery- and insurance-market equilibrium for the setup in Vasilev (2015a). When households in the setup are constrained to search for jobs only in one of the two sectors, in equilibrium there should be separate insurance firms: one for the public sector and another for the private sector, where each insurance company would pool the unemployment risk of the corresponding group of workers. This insurance market segmentation is an important new result in this literature, and is due to the presence of the double non-convexity, as well as the sorting effect of the sector-type shock in the model setup.

2. Model setup

The model follows Vasilev (2015a). The theoretical setup is a static economy, where agents face a non-convex decision in a two-sector economy. There is a large number of identical one-member households, indexed by i and distributed uniformly on the $[0,1]$ interval. The households will be assigned a sector "type," and after the type is revealed, each one decides whether to work in that sector or not. In the exposition below, we will suppress the index i to save on notation.

2.1 Households

Each household maximizes the following utility function

$$\max_{\{c,h^p,h^g\}} \left\{ \ln(c^\eta + S^\eta)^{\frac{1}{\eta}} + \alpha \ln(1 - h^p - h^g) \right\}, \tag{2.1}$$

where c, S, h^p, h^g denote household's private consumption, consumption of the public good, hours worked in the private sector, and hours worked in the government sector. The parameter $\alpha > 1$ measures the relative weight of leisure in the utility function. Total consumption is a Constant Elasticity of Substitution (CES) aggregation of private consumption and consumption of government services, where $\eta > 0$ measures the degree of substitutability between the two types of consumption.[2]

[2] The separability of consumption and leisure is not a crucial assumption for the results that follow. A more general, non-separable, utility representation, does not generate new results, while significantly complicates the algebraic derivations, and thus interferes with model tractability.

Each household is endowed with 1 unit of time that can be allocated to work in the private sector, work in the government sector, or leisure, so $h^p + h^g \leq 1$. Labor supply in each sector is discrete $h^p \in \{0, \overline{h}^p\}$, $h^g \in \{0, \overline{h}^g\}$, where $\overline{h}^p \leq 1$, $\overline{h}^g \leq 1$, and $\overline{h}^p + \overline{h}^g > 1$. In other words, working full-time in both sectors is infeasible, as it takes more than the total time available. Thus, the paper is consistent with Gomes (2014), who assumes that looking for a job will follow a "directed search" process: Each household decides in each period whether to go to the public or to the private sector (or, alternatively, is assigned a "sector type"). This process is stochastic and has two realizations. The probability of going to the private sector (or being a "private-sector type") is

$$q = \frac{H^p}{H^p + H^g}, \tag{2.2}$$

where uppercase letters denote aggregate quantities, i.e. H^p denotes aggregate hours in the private sector, and H^g are the aggregate hours worked in the public sector. Then the probability of being a public sector type is

$$1 - q = \frac{H^g}{H^p + H^g}. \tag{2.3}$$

This process is *i.i.d.* across individuals, so the Law of Large Number holds: At the aggregate level, q share of the households will be private sector type (and thus each household of this type would thus choose $h^g = 0$, as it searches for work only in the private sector), and $1 - q$ share will be public sector type (and thus each household of this type would thus choose $h^p = 0$, as it searches for work only in the public sector).[3] Once the particular sector-type is determined, each household decides on its labor supply accordingly. Note that the setup is quite general and allows for different wage rates per hour worked in the two sectors.

In addition to the work income, households hold shares in the private firm and receive profit share π, with $\int_0^1 \pi di = \Pi$.[4] Income is subject to a lump-sum tax t, where $\int_0^1 t di = T$. Therefore, each household's budget constraint is

$$c^j \leq w^j h^j + \pi - t, \quad j = p, g. \tag{2.4}$$

Households act competitively by taking the wage rates $\{w^p, w^g\}$, aggregate outcomes $\{S, H^p, H^g\}$ and lump-sum taxes $\{T\}$ as given. Each household chooses $\{c^j, h^p, h^g\}$ to maximize (2.1) s.t. (2.2)–(2.4).

[3] So the labor supply choice in a sector different from the type of the respective household is degenerate, as it will never be positive.

[4] This is a technical assumption which would guarantee a positive consumption to either of the two types, even if they choose not to work in their sector.

2.2 Firms

Next, there is a single firm producing a homogeneous final consumption good, which uses labor H^p as the only input. The production function is given by

$$Y = F(H^p),\ F' > 0,\ F'' < 0,\ F'(\overline{H}^p) = 0,$$

where the last assumption is imposed to proxy a capacity constraint. The firm takes $\{w^p\}$, aggregate outcomes $\{S, H^g\}$ and policy variable $\{T\}$ as given, and chooses $\{H^p\}$ to

$$\max_{H^p} F(H^p) - w^p H^p \quad \text{s.t } H^p \geq 0.$$

2.3 Government

The public authority hires H^g employees to provide public services, which are paid $w^g = \gamma w^p$, with $\gamma \geq 1$, as in the EU, average public sector wages feature a mark-up over private sector ones (Vasilev 2015b). The production function of non-market public services is as follows:

$$S = S(H^g),\ S' > 0,\ S'' < 0,\ S'(\overline{H}^g) = 0,$$

where the last assumption guarantees that not all "public-sector types" will work in the production of the public good. The public sector wage bill is financed by levying a lump-sum tax T on all households, or $w^g H^g = T$. The government takes H^g as given, and sets w^g, as a fixed gross mark-up above w^p, while T is residually chosen to ensure budget balance.

Vasilev (2015a) establishes that in equilibrium, given an initial realization of a type-specific shock, a fraction λ^p of the private-sector-type households would be working in the private sector, where c_w^p denotes consumption of those working, and c_n^p denotes consumption of those not working. Similarly, a fraction λ^g of the public-sector-type households would be working in the public sector and consuming c_w^g, while the public-sector types will be enjoying c_n^g. Alternatively, the workers would be participating in a sector-specific lottery with the proportions representing the probability of being selected for work. Conditional on the sector type, a household would receive the same income in expected terms.

Alternatively, we can introduce insurance markets, and allow households to buy insurance, which would allow them to equalize the actual income received, conditional on the sector-type. Given the difference in the wages and hours worked across sectors, segmented insurance markets are needed in order to provide actuarially fair insurance.

2.4 Insurance markets

Insurance markets is segmented, with one representative company per sector.[5] Insurance costs q^j per unit, $j = p, g$, and provides one unit of income if the household is not

[5] The insurance market segmentation is a direct effect of the discreteness of the labor supply in each sector and the sorting done by the sector-type shock.

working. We can think of insurance as bonds that pay out only in case the household is not chosen for work. Thus, household will also choose the quantity of insurance to purchase b^j, $j = p, g$. With sector types, the setup requires that the insurance market is segmented, with public sector insurance market insuring only public-sector-type households, and the private sector insurance market insuring only private-sector-type households.

Without segmentation, insurance will not be actuarially fair, one of the groups will face better odds versus price, the company will not be able to break even, and/or at least one type of households will not be able to buy full insurance, which would completely smooth consumption across employment states, given the non-convexity constraint of labor supply.

As pointed out in Hansen (1985), the plausibility of this insurance market segmentation result depends crucially on the fact that probabilities λ^p and λ^g are perfectly observable to everyone, and that the contracts written are perfectly enforceable. Also, who has won and who has lost the lottery is assumed to be perfect knowledge. Lastly, everyone will always announce truthfully the same λ^p (λ^g) to the private (public) firm and the private-sector (public-sector) insurance company.

2.4.1 Private-sector insurance company

The private-sector insurance company maximizes profit. The company only services private-sector types. It receives revenue if a private-sector-type household is working and makes payment if it is not. More specifically, the proportion of people working in the private sector contribute towards the unemployment benefits pool, which are then distributed of benefits to the unemployed in that sector. The amount of insurance sold in the private sector is a solution to the following problem: Taking $q^{p*}(i)$ as given, $b^{p*}(i)$ solves

$$\max_{b^p} \lambda^{p*}(i) q^{p*}(i) b^p - [1 - \lambda^{p*}(i)] b^p.$$

With free entry profits are zero, hence

$$\lambda^{p*}(i) q^{p*}(i) b^p - [1 - \lambda^{p*}(i)] b^p = 0.$$

This condition implicitly clears the insurance market for each individual in the private sector.

2.4.2 Public-sector insurance company

The public-sector insurance company also maximizes profit. The company only services public-sector types. It receives revenue if a public-sector-type household is working and makes payment if it is not. More specifically, the proportion of people working in the public sector contribute towards the unemployment benefits pool, which are then distributed of benefits to the unemployed in that sector. The amount of insurance sold in the public sector is a solution to the following problem: Taking $q^{g*}(j)$ as given,

$b^{g*}(j)$ solves

$$\max_{b^g} \lambda^{g*}(j)q^{g*}(j)b^g - [1-\lambda^{g*}(j)]b^g$$

With free entry profits of the insurance company operating in the public sector are also zero since

$$\lambda^{g*}(j)q^{g*}(j)b^g - [1-\lambda^{g*}(j)]b^g = 0.$$

This implicitly clears the insurance market for each individual of a public sector type.

In the next section, the equilibrium with lotteries and no insurance markets is presented and discussed first, and then the setup is extended to incorporate a regime with insurance.

3. Decentralized Competitive Equilibrium (DCE) with lotteries

3.1 Definition of the DCE with lotteries

A competitive equilibrium with lotteries in the private and public sector for this economy is a list:

$$\left(c_w^{p*}(i), c_n^{p*}(i), \lambda^{p*}(i)\right), \left(c_w^{g*}(j), c_n^{g*}(g), \lambda^{g*}(j)\right), h^{f*}, w^{p*}, w^{g*}, p^*, \pi^*$$

s.t.

(i) Private-sector consumer maximization – taking w^{p*}, p^{p*}, π^* as given, for each private-sector type household i, $c_w^{p*}(i)$, $c_n^{p*}(i)$, $\lambda^{p*}(i)$ solve:[6]

$$\max_{\lambda^p, c_w^p, c_n^p} \lambda^p(i)\Big\{ \ln[(c_w^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\bar{h}^p)\Big\} + \\ +(1-\lambda^p(i))\Big\{ \ln[(c_n^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1)\Big\}$$

$$\text{s.t.} \quad p^*[\lambda^p(i)c_w^p + (1-\lambda^p(i))c_n^p] = w^{p*}\bar{h}^p\lambda^p(i) + \pi^* - t, \\ c_w^p \geq 0,\ c_n^p \geq 0,\ 0 < \lambda^p(i) < 1.$$

(ii) Public-sector consumer maximization – taking w^{g*}, p^{p*}, π^* as given, for each public-sector type household j, $c_w^{g*}(j)$, $c_n^{g*}(j)$, $\lambda^{g*}(i)$ solve

$$\max_{\lambda^g, c_w^g, c_n^g} \lambda^g(j)\Big\{ \ln[(c_w^g)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\bar{h}^g)\Big\} + \\ +(1-\lambda^g(j))\Big\{ \ln[(c_n^g)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1)\Big\}$$

[6] Note that now when they trade lotteries the outcome is no longer deterministic. Now consumers maximize expected utility, i.e. if a private sector type is chosen to work with probability λ^p, that individual will get expected income $\lambda^p w^p \bar{h}^p$.

$$\text{s.t.} \quad p^*[\lambda^g(j)c_w^g + (1-\lambda^g(j))c_n^g] = w^{g*}\overline{h}^g\lambda^g(j) + \pi^* - t,$$
$$c_w^g \geq 0,\ c_n^g \geq 0,\ 0 < \lambda^g(j) < 1.$$

(iii) Firm maximization – taking p^*, w^{p*} as given, h^{f*} solves

$$\max_h \quad p^* f(h) - w^{p*}h$$
$$\text{s.t.} \quad h \geq 0,$$

and

$$\pi^* = p^* f(h^{f*}) - w^{p*}h^{f*}.$$

(iv) Government – taking p^*, w^{p*}, and π^* as given, government provides public services according to the following production function

$$S = S(\lambda^{g*}\overline{h}^{g*})$$

The government sets $w^{g*} = \gamma w^{p*}$. Finally, T is residually set to ensure

$$w^{g*}\lambda^{g*}\overline{h}^g = T.$$

(v) Market clearing:

$$\int_i \lambda^{p*}(i)\overline{h}^p di = h^{f*},$$
$$\int_i \left[\lambda^{p*}(i)c_w^{p*}(i) + (1-\lambda^{p*}(i))c_n^{p*}(i)\right] di + \int_j \left[\lambda^{g*}(j)c_w^{g*}(j) + (1-\lambda^{g*}(j))c_n^{g*}(j)\right] dj = f(h^{f*}).$$

3.2 Characterization of the DCE with lotteries

Private-sector types problem is:

$$L = \lambda^p(i)\left\{\ln[(c_w^p)^\eta + S^\eta]^{1/\eta} + \alpha\ln(1-\overline{h}^p)\right\} + (1-\lambda^p(i))\left\{\ln[(c_n^p)^\eta + S^\eta]^{1/\eta}\right\}$$
$$-\mu\left\{p^*\lambda^p(i)c_w^p + p^*(1-\lambda^p(i))c_n^p - w^{p*}\overline{h}^p\lambda^p(i) - \pi^* + t\right\}$$

FOCs:

$$c_w^p: \qquad \lambda^p(i)\frac{1}{[(c_w^p)^\eta + S^\eta]}(c_w^p)^{\eta-1} = \mu p^* \lambda^p(i) \tag{3.1}$$

$$c_n^p: \qquad (1-\lambda^p(i))\frac{1}{[(c_n^p)^\eta + S^\eta]}(c_n^p)^{\eta-1} = \mu p^*(1-\lambda^p(i)) \tag{3.2}$$

$$\begin{aligned} \lambda^p(i): \qquad & \left\{\ln[(c_w^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\overline{h}^p)\right\} - \left\{\ln[(c_n^p)^\eta + S^\eta]^{1/\eta}\right\} \\ & -\mu\left\{p^* c_w^p - p^* c_n^p - w^{p*}\overline{h}^p\right\} = 0 \end{aligned} \tag{3.3}$$

(3.1) and (3.2) show that $c_w^p = c_n^p$, $\forall i$. Also, $\lambda^p(i) = \lambda^p$, $\forall i$. Then (3.3) simplifies to

$$\alpha \ln(1-\overline{h}^p) = -\mu w^{p*}\overline{h}^p.$$

Hence,

$$w^{p*} = f'(\lambda^{p*}\overline{h}^{p*}) = \frac{\alpha \ln(1-\overline{h}^p)[(c_w^p)^\eta + S^\eta]}{(c_w^p)^{\eta-1}\overline{h}^p}.$$

This equation is a discrete version of the marginal product of labor equals the marginal rate of substitution. It implicitly characterizes the optimal λ^p.

Public-sector types problem:

$$\begin{aligned} L = \lambda^g(j)&\left\{\ln[(c_w^g)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\overline{h}^g)\right\} + (1-\lambda^g(j))\left\{\ln[(c_n^g)^\eta + S^\eta]^{1/\eta}\right\} \\ & -\nu\left\{p^*\lambda^g(j)c_w^g + p^*(1-\lambda^g(j))c_n^g - w^{g*}\overline{h}^g\lambda^g(j) - \pi^* + t\right\} \end{aligned}$$

FOCs:

$$c_w^g: \qquad \lambda^g(j)\frac{1}{[(c_w^g)^\eta + S^\eta]}(c_w^g)^{\eta-1} = \nu p^* \lambda^g(j) \tag{3.4}$$

$$c_n^g: \qquad (1-\lambda^g(j))\frac{1}{[(c_n^g)^\eta + S^\eta]}(c_n^g)^{\eta-1} = \nu p^*(1-\lambda^g(j)) \tag{3.5}$$

$$\begin{aligned} \lambda^g(j): \qquad & \left\{\ln[(c_w^g)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\overline{h}^g)\right\} - \left\{\ln[(c_n^g)^\eta + S^\eta]^{1/\eta}\right\} \\ & -\nu\left\{p^* c_w^g - p^* c_n^g - w^{g*}\overline{h}^g\right\} = 0 \end{aligned} \tag{3.6}$$

(3.4) and (3.5) show that $c_w^g = c_n^g$, $\forall j$. Also, $\lambda^g(j) = \lambda^g$, $\forall j$. Then (3.6) simplifies to

$$\alpha \ln(1-\overline{h}^g) = -\nu w^{p*}\overline{h}^p.$$

Hence,

$$w^{g*} = \gamma f'(\lambda^{p*}\overline{h}^{p*}) = \frac{\alpha \ln(1-\overline{h}^{g})[(c_w^g)^\eta + S^\eta]}{(c_w^g)^{\eta-1}\overline{h}^{g}}.$$

This equation is a also the discrete version of the marginal product of labor equals the marginal rate of substitution. In this case it implicitly characterizes the optimal λ^g. Note that it is optimal for the benevolent government point of view to choose randomly λ^p, λ^g and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness is achieved.

Since a household of either type can be chosen to work with some probability, the households are exposed to risk. Hence it would be optimal to have insurance. The government can then sell employment lotteries, and individuals will buy insurance to cover the risk of being unemployed (not being chosen for work). With insurance, however, the employer pays wage to individuals only if they work. That is, instead of working with expected income, we will work with actual income. This allows to extend the commodity space in the model framework and include insurance markets.

4. Decentralized Competitive Equilibrium with lotteries and insurance markets

4.1 Definition of the DCE with insurance markets

A competitive equilibrium with lotteries and unemployment insurance is a list

$$\left(c_w^{p*}(i), c_n^{p*}(i), \lambda^{p*}(i), b^{p*}(i)\right), \left(c_w^{g*}(j), c_n^{g*}(g), \lambda^{g*}(j), b^{p*}(j)\right), h^{f*}, w^{p*}, w^{g*}, p^*, q^{p*}, q^{g*}, \pi^*$$

s.t.

(i) Private-sector-type household maximization – taking w^{p*}, p^{p*}, π^* as given, for each private-sector type household i, $c_w^{p*}(i), c_n^{p*}(i), \lambda^{p*}(i), b^*(i)$ solve

$$\max_{\lambda^p, c_w^p, c_n^p} \lambda^p(i)\Big\{ \ln[(c_w^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\overline{h}^p)\Big\} +$$
$$+(1-\lambda^p(i))\Big\{ \ln[(c_n^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1)\Big\}$$

$$\text{s.t.} \qquad p^* c_w^p + b^p q^{p*}(i) = w^{p*}\overline{h}^p + \pi^*,$$
$$p^* c_n^p = b^p + \pi^*,$$
$$c_w^p \geq 0,\ c_n^p \geq 0,\ 0 < \lambda < 1,$$

or

$$p^* c_w^p + p^* q^{p*} c_n^p = w^{p*}\overline{h}^p + (1+\pi^*) q^{p*}.$$

Foe each household in the private sector, there are two states: a household is buying unemployment insurance when working, receiving a payout when not working, hence in equilibrium $b^{p*} = \lambda^{p*} w^{p*} \overline{h}^{p*}$.

(ii) Public-sector-type household maximization – taking w^{g*}, p^{p*}, π^* as given, for each public-sector-type household j, $c_w^{g*}(j), c_n^{g*}(j), \lambda^{g*}(j), b^{g*}(j)$ solve

$$\max_{\lambda^g, c_w^g, c_n^g} \lambda^g(j) \Big\{ \ln[(c_w^g)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1 - \overline{h}^g) \Big\} + $$
$$+ (1 - \lambda^g(j)) \Big\{ \ln[(c_n^g)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1) \Big\}$$

$$\begin{aligned} \text{s.t.} \qquad & p^* c_w^g + b^g q^{g*}(i) = w^{g*} \overline{h}^g + \pi^*, \\ & p^* c_n^g = b^g + \pi^*, \\ & c_w^g \geq 0,\ c_n^g \geq 0,\ 0 < \lambda^g < 1, \end{aligned}$$

or

$$p^* c_w^p + p^* q^{p*} c_n^p = w^{p*} \overline{h}^p + (1 + \pi^*) q^{p*}.$$

Foe each household in the public sector, there are two states: a household is buying unemployment insurance when working, receiving a payout when not working, hence in equilibrium $b^{g*} = \lambda^{g*} w^{g*} \overline{h}^{g*}$.

(iii) Firm maximization – taking p^*, w^* as given, h^{f*} solves

$$\begin{aligned} \max_h \qquad & p^* f(h) - w^{p*} h, \\ \text{s.t.} \qquad & h \geq 0, \end{aligned}$$

and

$$\pi^* = p^* f(h^{f*}) - w^{p*} h^{f*}.$$

(iv) Insurance companies. Insurance markets is segmented, with one company per sector.

(a) Private sector. Taking $q^{p*}(i)$ as given, $b^{p*}(i)$ solves

$$\max_{b^p} \lambda^{p*}(i) q^{p*}(i) b^p - (1 - \lambda^{p*}) b^p$$

With free entry profits are zero, hence

$$\lambda^{p*}(i) q^{p*}(i) b^p - (1 - \lambda^{p*}(i)) b^p = 0.$$

This implicitly clears the insurance market for each individual in the private sector.

(b) Public sector. Taking $q^{g*}(j)$ as given, $b^{g*}(j)$ solves

$$\max_{b^g} \lambda^{g*}(j)q^{g*}(j)b^g - (1-\lambda^{g*}(j))b^g$$

With free entry profits of the insurance company operating in the public sector are also zero since

$$\lambda^{g*}(j)q^{g*}(j)b^g - (1-\lambda^{g*}(j))b^g = 0.$$

This implicitly clears the insurance market for each individual of a public sector type.

(v) Government – taking p^*, w^{p*}, and π^* as given, government provides public services according to $S = S(\lambda^{g*}\overline{h}^{g*})$. The government sets $w^{g*} = \gamma w^{p*}$, and taxes T are residually set to ensure

$$w^{g*}\lambda^{g*}\overline{h}^g = T.$$

(vi) Market clearing.

$$\int_i \lambda^{p*}(i)\overline{h}^p di = h^{f*},$$

$$\int_i [\lambda^{p*}(i)c_w^{p*}(i) + (1-\lambda^{p*}(i))c_n^{p*}(i)]di + \int_j [\lambda^{g*}(j)c_w^{g*}(j) + (1-\lambda^{g*}(j))c_n^{g*}(j)]dj = f(h^{f*}).$$

4.2 Characterization of the DCE with insurance markets

Private sector consumer problem:

$$\max_{\lambda^p, c_w^p, c_n^p, b^p(i)} \lambda^p(i)\Big\{\ln[(c_w^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1-\overline{h}^p)\Big\} +$$
$$+(1-\lambda^p(i))\Big\{\ln[(c_n^p)^\eta + S^\eta]^{1/\eta} + \alpha \ln(1)\Big\}$$
$$\text{s.t.} \qquad p^* c_w^p + p^* q^{p*} c_n^p = w^{p*}\overline{h}^p + \pi^* + q^{p*}\pi^*.$$

Normalize $p^* = 1$.

$$c_w^p: \qquad \lambda^p \frac{(c_w^p)^{\eta-1}}{[(c_w^p)^\eta + S^\eta]} = p\mu$$
$$c_n^p: \qquad (1-\lambda^p)\frac{(c_n^p)^{\eta-1}}{[(c_n^p)^\eta + S^\eta]} = pq^p\mu$$

Optimal λ^p ($\lambda^p(i) = \lambda^p$, $\forall j$) is implicitly characterized by the zero-profit condition from the private sector insurance company:

$$\frac{\lambda^p}{1-\lambda^p} = \frac{1}{q^p} \tag{4.1}$$

The price of insurance depends on probability of the event you are insuring against. We cannot force $q^{p*}(i) = q^{p*}$ although ex post that would indeed be the case. For the insurance firms, profits are linear in q^p. This implies that profits cannot be positive or negative in equilibrium. Zero profits in the private sector insurance market then mean $q^p = \frac{1-\lambda^p}{\lambda^p}$. A common interpretation for both insurance companies is that this price of the insurance is the odds ratio, or the ratio of probabilities of the two events.

Combining then with the FOCs for state-contingent consumption, we obtain that $c_w^p = c_n^p$, $\forall i$. That is, private-sector-type households buy full insurance to smooth consumption perfectly.

Similarly, for the public sector consumers:

$$\max_{\lambda^g, c_w^g, c_n^g}, b^g(j)\lambda^g(j)\Big\{\ln[(c_w^g)^\eta + S^\eta]^{1/\eta} + \alpha\ln(1-\overline{h}^g)\Big\} +$$

$$+(1-\lambda^g(j))\Big\{\ln[(c_n^g)^\eta + S^\eta]^{1/\eta} + \alpha\ln(1)\Big\}$$

$$\text{s.t.} \qquad c_w^g + q^{g*}c_n^g = w^{g*}\overline{h}^g + \pi^* + q^{g*}\pi^*.$$

$$c_w^g: \qquad \lambda^g \frac{(c_w^g)^{\eta-1}}{[(c_w^g)^\eta + S^\eta]} = pv$$

$$c_n^g: \qquad (1-\lambda^g)\frac{(c_n^g)^{\eta-1}}{[(c_n^g)^\eta + S^\eta]} = pq^g v$$

Optimal λ^g ($\lambda^g(j) = \lambda^g$, $\forall j$) is implicitly characterized by the zero-profit condition from the public sector insurance company:

$$\frac{\lambda^g}{1-\lambda^g} = \frac{1}{q^g}$$

Combining then with the FOCs for state-contingent consumption, we obtain that $c_w^g = c_n^g$, $\forall j$. Also, $\lambda^g(j) = \lambda^g$, $\forall j$. That is, public-sector-type households buy also full insurance to equalize consumption in the two states (employed *vs.* unemployed). In particular, when income is stochastic, i.e., it is uncertain whether the individual will be employed, we need insurance markets for each sector type. In this economy there is no uncertainty (after the types are revealed) but it is optimal to introduce insurance markets. This is because of the non-convexity of the choice set, which is similar to having incomplete markets. Lotteries can then be introduced to achieve market completeness. Therefore, randomization may be optimal in a non-convex environment even though there is no aggregate uncertainty.

5. Conclusions

This note describes the lottery and insurance market equilibrium in an economy with both private and public sector jobs and non-convex labor supply decision faced by the workers. In addition, when households are constrained to search for jobs only in a certain sector, the framework requires that there should be separate insurance markets: public and private sector one, which would pool the risk of the corresponding group of workers. In equilibrium, conditional on the sector-type, each household would fully insure against the uncertainty in terms of the employment status (but cannot insure against the "type" shock). The unemployment insurance market segmentation is a direct effect of the discreteness of the labor supply in each sector and the sorting done by the sector-type shock.

The plausibility of the result derived in the paper depends crucially on the fact that probabilities λ^p and λ^g are perfectly observable to everyone, and that the contracts written are perfectly enforceable. Also, who has won and who has lost the lottery is assumed to be perfect knowledge. Lastly, everyone will always announce truthfully the same λ^p (λ^g) to the private (public) firm and the private-sector (public-sector) insurance company. Therefore, whether and how this insurance-market segmentation can be implemented in reality is not entirely clear at this point.

Acknowledgement The author would like to thank the Editor and an anonymous referee for the valuable comments.

References

Cooley, T. and Prescott, E. (1995). Economic Growth and Business Cycles. In Cooley, T. (ed.), *Frontiers of Business Cycle Research*. Princeton NJ, Princeton University Press.

Gomes, P. (2014). Optimal Public Sector Wages. *The Economic Journal*, 125, 1425–1451

Hansen, G. (1985). Indivisible Labor and the Business Cycle. *Journal of Monetary Economics*, 16, 309–328.

Kydland, F. (1995). Business Cycles and Aggregate Labor Market Fluctuations. In Cooley, T. (ed.), *Frontiers of Business Cycle Research*. Princeton NJ, Princeton University Press.

Rogerson, R. (1988). Indivisible Lotteries, Lotteries and Equilibrium. *Journal of Monetary Economics*, 21, 3–16.

Vasilev, A. (2015a). Aggregation with a Double Non-Convex Labor Supply Decision: Indivisible Private- and Public-Sector Hours. Submitted to the *Journal of Economic Development*, under review.

Vasilev, A. (2015b). Macroeconomic Effects of Public-Sector Unions. *LABOUR*, 29(2), 101–126.