Czech Economic Review 9 (2015) 36–48
Acta Universitatis Carolinae Oeconomica

Participation and Solidarity in Redistribution Mechanisms

José-Manuel Giménez-Gómez[*] and Josep E. Peris[**]

Received 24 February 2015; Accepted 10 June 2015

Abstract Following Bossert (1995), we consider a model where personal income depends on two different characteristics: skills and effort. Luttens (2010) introduces claims that individuals have over aggregate income and that only depend on the effort they exert. Moreover, he proposes redistribution mechanisms in which *solidarity* is based on changes in a lower bound on what every individual deserves according to these claims: the so-called *minimal rights* (O'Neill 1982). A debatable consequence in one of Luttens' mechanisms is that "the poorest individuals might up with a negative income" (Luttens 2010); that is, this mechanism does not satisfy *participation*, which turns out to be incompatible with *claims feasibility*, under Luttens' assumptions. We present a new solidarity axiom that is compatible both with *participation* and *claims feasibility*, and we provide a mechanism satisfying these properties and our *new additive solidarity axiom*. Moreover, our mechanism satisfies additional properties, as *priority*, or *respect of minimal rights*.

Keywords Redistribution mechanism, minimal rights, solidarity, participation, claims feasibility
JEL classification C71, D63, D71

1. Introduction

We suppose that inequalities in welfare among individuals in a society are determined by two different factors or characteristics: *skills* and *effort*. The difference between these characteristics is that individuals are completely responsible for the inequalities due to differences in effort, that do not deserve compensation (effort reflects, for instance, the number of hours that a person decides to work). Nevertheless, there are other circumstances, which are beyond the control of the individuals, that deserve compensation (different innate skills or talents, economic status, historical inequality due to race, gender etc.).

The aim of fair income redistribution is to guarantee an equal income for individuals exerting the same effort (*the principle of compensation*) and to perform equal income transfers to individuals with equal skills (*the principle of natural reward*). It is well known that, in many contexts, a redistribution mechanism satisfying both the principle of compensation and the principle of natural reward simultaneously does not exist. As a result, the literature has concentrated on dealing with such trade-off between both principles. Notably, most contributions have opted for a weakening of the principle of natural reward (see, for instance, Fleurbaey 1994; Bossert 1995; Bossert and Fleurbaey

[*] Universitat Rovira i Virgili, Dep. d'Economia and CREIP, Av. Universitat 1, 43204 Reus, Spain. Tel.: (+34) 977 759 850, E-mail: josemanuel.gimenez@urv.cat.
[**] Corresponding author. Universitat d'Alacant, Dep. de Mètodes Quantitatius i Teoria Econòmica, 03080 Alacant, Spain. E-mail: peris@ua.es.

1996; Iturbe-Ormaetxe 1997; Tungodden 2005). Other approaches have opted for strengthening compensation with respect to the principle of solidarity, a principle with a long tradition in the theory of justice.[1] In accordance, Bossert and Fleurbaey (1996) and Iturbe-Ormaetxe (1997) present two modified versions of the solidarity axiom (*additive solidarity* and *multiplicative solidarity*) to characterize the egalitarian equivalent mechanisms (Pazner and Schmeidler 1978) and the family of proportionally adjusted equivalent mechanisms (Iturbe-Ormaetxe 1997), respectively. A central notion on fairness is the *No-Envy* property, suggested by Foley (1967) and analyzed by Kolm (1972), Panzer and Schmeidler (1974) and Varian (1974).

We use the quasi-linear model developed in Bossert (1995), with utility functions taking the form

$$u_i = x_i + v(y_i, z_i),$$

where $v(y_i, z_i)$ is described as agent pre-tax income, x_i is an income transfer, and u_i is the final income after redistribution. As usual in this model, we consider that the total amount to be distributed is $\Omega = 0$; that is, we consider a *redistribution problem*.

An important feature of this model is that the redistribution of resources is based only on the set of characteristics that deserve compensation. Furthermore, such compensation is assigned according to a *solidarity* basis (Rawls 1971): *changes in these characteristics should affect each individual's final utility in the same direction*. Accordingly, Bossert (1995) proposes the property of *additive solidarity*, which is based on the idea that individuals should benefit equally from variations in the skills profile.

In a recent paper, Luttens (2010) includes a new element into the model, by defining a *claims* function that depends on the individual's effort z but does not depend on the individual's skill y, "hence, two individuals with identical effort, but different skills (different pre-tax incomes) have identical claims in the redistribution problem" (Luttens 2010). Within this approach, Luttens makes a bridge between the conflicting claims literature and the Bossert's taxation model (Bossert 1995) with quasi-linear preferences previously mentioned. Moreover, he proposes a lower bound in the individuals' welfare based on their claims function, namely the *minimal rights lower bound*,[2] and defines a strengthening of the additive solidarity principle: *an income gain (loss), generated by a change in the skills profile, is shared on the basis of the information contained in changes of this lower bound*.

The redistribution mechanism proposed by Luttens (2010) fails to respect the minimal rights lower bound on which it is based; that is, for some individuals the income after redistribution might be lower than her *minimal right*. Moreover, as the author suggests, "a debatable property is that the poorest individuals might end up with a negative income after redistribution when R (the aggregate income) is sufficiently low." This property corresponding to the notion of *participation* (Maniquet 1998) "captures the idea of protecting high-skilled agents in the sense that the low-skilled agent compensation should not be carried out by imposing on the former agents so long labor time that they end up worse off than if they withdrew from the economy" (see also

[1] See Fleurbaey and Maniquet (2011) for a comprehensive summary of this literature. We follow this paper for notation and definitions.

[2] This bound was introduced by O'Neill (1982) in the context of claims problems.

Fleurbaey and Maniquet 2011). In other contexts, this non-negativity condition has also been adopted. For instance, "in production models, non-negativity (*participation*) takes care of the "slavery" problem, by avoiding situations in which an agent would rather opt out of the economy than participate in the production process" (Fleurbaey 2008). Given a change in the skill profile, a planner can perform a redistribution by following a *solidarity* property, but this should not come at the cost of an agent ending up with a negative income after redistribution. Once an agent ends up with a zero income after redistribution, she no longer needs to take part in performing solidarity. Luttens (2010), in order to solve this problem, defines an alternative mechanism satisfying *participation*, at the expense of losing *claims feasibility*.

We are interested in keeping both properties along with the axiom of *respect of the minimal rights*. Then, we propose a refinement of Luttens' mechanisms which makes compatible *participation* and *claims feasibility* by weakening the *solidarity* condition. Note that *minimal rights* suppose a very weak notion of guarantee: it requires that each individual receives at least what is left of the resources after the other claims have been fully compensated, or zero if this amount is negative. So, if the claims are high enough with respect to the aggregate income, no individual guarantee exists at all.

The paper is organized as follows. In Section 2, we present the model and introduce the basic definitions. Section 3 proposes and characterizes our respect of minimal rights-based egalitarian mechanism. Some final remarks close the paper in Section 4. An appendix gathers the proof of our characterization result and the independence of the axioms used in this characterization.

2. The model

2.1 Fair monetary compensation model

Let us denote by $N = \{1,\dots,n\}$ the finite population of size $n \geq 2$. Individuals are distinguished by two characteristics: *skill* and *effort*. Differences in skill elicit compensation. The individual's skill is represented by a real non-negative number $y \in \mathbb{Y}$, where $\mathbb{Y}$ is an interval of $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. The skills profile is the vector $y_N = (y_1,\dots,y_n) \in \mathbb{Y}^n$. The effort is also denoted by a non-negative real number $z \in \mathbb{Z}$, where $\mathbb{Z}$ is an interval of $\mathbb{R}_+$. The effort profile is $z_N = (z_1,\dots,z_n) \in \mathbb{Z}^n$. So, each individual is identified by the pair of non-negative real numbers $(x_i, y_i) \in \mathbb{Y} \times \mathbb{Z}$, specifying her skill and effort, respectively. Without loss of generality, throughout the work we assume that individuals are ranked with respect to the effort they exert: $z_1 \geq z_2 \geq \ldots \geq z_n$. An economy consists of the pair of skill and effort profiles, $e = (y_N, z_N) \in \mathbb{Y}^n \times \mathbb{Z}^n$. Let $\mathcal{E}$ denote the set of economies, $\mathcal{E} \subseteq \mathbb{Y}^n \times \mathbb{Z}^n$.

Given an economy $e = (y_N, z_N) \in \mathcal{E}$, a *pre-tax income* function (identical for all individuals), $v : \mathbb{Y} \times \mathbb{Z} \to \mathbb{R}_+$, associates to each individual (y_i, z_i) a monetary income $v(y_i, z_i)$ that depends on her skill and effort. It is supposed that function v is strictly increasing in y. The total sum of pre-tax incomes is denoted by $R = \sum_{i \in N} v(y_i, z_i)$.

Differences in individuals' skills are compensated by an amount x_i of a transferable resource (money). Differences in effort do not elicit compensation. An allocation $x_N = (x_1,\dots,x_n) \in \mathbb{R}^n$ is the vector defined by transferable resources x_i. We assume that the total amount to be distributed is $\Omega = 0$, so that we are looking at a redistribution problem (total subsidies coincide with total taxes). Then, an allocation is *feasible* whenever $\sum_{i \in N} x_i = 0$.

We assume, as in Luttens (2010), that individuals, because of the effort they exert, have some *claim* on the total pre-tax income R. Let $g : \mathbb{Z} \to \mathbb{R}_{++}$ be the *claims function* that assigns to each individual (y_i, z_i) a claim $g(z_i)$ that depends on the individual's effort only. We assume that function $g(z)$ is continuous and strictly increasing in z. We denote the total sum of claims by $C = \sum_{i \in N} g(z_i)$, and $C_{-i} = \sum_{j \neq i \in N} g(z_j)$. The redistribution problem will be a conflicting claims problem whenever $C > R$. Let us denote the claims vector of an economy $e = (y_N, z_N)$ by $\hat{\mathbf{g}} = (g(z_1), g(z_2), \ldots, g(z_n))$.

A (*redistribution*) mechanism is a function $S : \mathcal{E} \times \mathbb{R}^n \to \mathbb{R}^n$ such that for all $e \in \mathcal{E}$, and any claims vector $\hat{\mathbf{g}}$, $S(e, \hat{\mathbf{g}})$ is a feasible allocation, that is

$$\sum_{i \in N} S_i(e, \hat{\mathbf{g}}) = 0.$$

It is assumed, as in Bossert (1995), that individuals preferences are characterized by (quasi-linear) utility functions, $u : \mathbb{R} \times \mathbb{Y} \times \mathbb{Z} \to \mathbb{R}$, which are defined as follows:

$$u(x_i, y_i, z_i) = x_i + v(y_i, z_i).$$

Utility represents the final income after redistribution. It is clear that, as $\sum_{i \in N} x_i = 0$,

$$\sum_{i \in N} u(x_i, y_i, x_i) = \sum_{i \in N} v(y_i, z_i) = R.$$

As the compensation x_i each agent receives will depend on the claims vector, we shall denote the utility function of individual (y_i, z_i) by $u_i(e, \hat{\mathbf{g}})$,

$$u_i(e, \hat{\mathbf{g}}) = x_i(\hat{\mathbf{g}}) + v(y_i, z_i).$$

We now introduce some notation that will be helpful. Given two economies $e = (y_N, z_N)$ and $e' = (y'_N, z_N)$, which only differ in skills profiles, and any claims vector $\hat{\mathbf{g}}$, changes in any function or variable are denoted by the difference operator Δ. Then, $\Delta h = h(e', \hat{\mathbf{g}}) - h(e, \hat{\mathbf{g}})$. Note that, since the effort is the same in both economies, the claims vector also coincides. This notation will be used to represent changes in the utility function, u, the minimal rights vector, m, as well as changes in the total pre-tax income, R (although R does not depends on the claims vector $\hat{\mathbf{g}}$).

2.2 Axioms

Before introducing the axioms, we provide the definition of the minimal rights lower bound (O'Neill 1982). This bound guarantees to each individual the amount that is left when the rest of the individuals have received their claim, or zero if this amount is negative. Associated with this bound, *respect of minimal rights* states that each individual should receive at least her minimal right (this axiom is a consequence of efficiency, non-negativity and claims boundedness combined (Thomson 2003)).

Definition 1. *Minimal rights* (O'Neill 1982). For each economy $e = (y_N, z_N) \in \mathcal{E}$, and each claims vector $\hat{\mathbf{g}}$, the minimal rights vector, $m \in \mathbb{R}^n_+$, is defined by $m_i = m_i(e, \hat{\mathbf{g}}) = \min\left\{g(z_i), [R - C_{-i}]_+\right\}$, $i \in N$, where $[a]_+ = \max\{0, a\}$.

Axiom 1. *Respect of minimal rights* (*RMR*). For each economy $e = (y_N, z_N) \in \mathcal{E}$, each claims vector $\hat{\mathbf{g}}$, and each $i \in N$, $u_i(e, \hat{\mathbf{g}}) \geq m_i(e, \hat{\mathbf{g}})$.

The following axiom, *participation*, states that no individual can incur losses, i.e. when R converges to zero, all incomes should also converge to zero. Moreover, note that before redistribution the pre-tax income of each individual $v_i = v(y_i, z_i)$ is non-negative, so after redistribution this condition should be maintained. Note that this property is implied by the axiom *respect of minimal rights*.

Axiom 2. *Participation* (*P*, Maniquet 1998). For each economy $e = (y_N, z_N) \in \mathcal{E}$, each claims vector $\hat{\mathbf{g}}$, and each $i \in N$, $u_i(e, \hat{\mathbf{g}}) \geq 0$.

Next, we present the axioms characterizing our mechanism. The first one, *claims feasibility* is a standard assumption which requires that when the aggregate pre-tax income (resources) equals the aggregate claim, then each individual's utility equals her claim.

Axiom 3. *Claims Feasibility* (*CF*). For each economy $e = (y_N, z_N) \in \mathcal{E}$, and each claims vector $\hat{\mathbf{g}}$, if $R = C$, then $u_i(e, \hat{\mathbf{g}}) = g(z_i)$ for all $i \in N$.

The following property (Luttens 2010) requires an equal treatment of two individuals in the allocation of the extra resources when their minimal rights change equally.

Definition 2. *Additive Solidarity for equal changes in minimal rights* (AS^*, Luttens 2010). Given two economies $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, that only differ in skills profiles, and a claims vector $\hat{\mathbf{g}}$, if $\Delta m_i = \Delta m_j$, then $\Delta u_i = \Delta u_j$.

Finally, the following axiom establishes that the changes in the resources should be shared among those individuals with changes in their minimal rights.

Axiom 4. *Priority* (*PRI*, Luttens 2010). Given two economies $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, that only differ in skills profiles, and a claims vector $\hat{\mathbf{g}}$, if $N_1 = \{i \in N : \Delta m_i \neq 0\} \neq \varnothing$, and $\Delta m_i = \Delta R$, for all $i \in N_1$, then $\sum_{i \in N_1} \Delta u_i = \Delta R$; or, equivalently, $\Delta u_j = 0$, for all $j \notin N_1$.

Luttens (2010) proposes and characterizes two different mechanisms. One of them with the axioms AS^*, *PRI* (he names both axioms together as *minimal rights-based solidarity*) and *CF*; the other mechanism with AS^*, *PRI* and *P*. As a consequence, *minimal rights-based solidarity*, *claims feasibility* and *participation* are incompatible axioms. So, when imposing Luttens' *minimal rights-based solidarity* together with *claims feasibility*, a redistribution mechanism also fails the *respect minimal rights* axiom. Luttens argues that "this incompatibility is due to AS^* rather than *priority*." We agree on this argument and in order to obtain compatibility we just modify the AS^* axiom.

The following example will be useful to better observe what happens with the *minimal rights-based solidarity* axioms when *participation* is required.

Example 1. Let us consider an economy e, with $n = 4$ individuals such that $\hat{\mathbf{g}} = (80,70,50,30)$. As previously mentioned, *participation* implies $u_i = 0$ for all i, when $R' = 0$. In this case, the *minimal rights* vector is $m = (0,0,0,0)$. If, due to a change in the skills profile, we now have $R = 150$, then all *minimal rights* are still null, so axiom AS^* implies an equal sharing of the extra resources, that is $u_i = 37.5$ for all i. Then, individual 4 ends up at a welfare level that is above what she deserves (her *claim*) and the other individuals are below their claims. Thus, *claims feasibility* is not met.

What is the problem in the above example? In our opinion, the "problem" occurs when the *minimal right* equals to zero, and it is originated because of the $[\,]_+$ operator that appears in the definition of *minimal rights*. To see this, observe that whenever $R \leq 150$ all minimal rights are null, but there is a significant difference before applying the $[\,]_+$ operator:

$$m_1 = \min\left\{80, [0]_+\right\} = 0, \qquad m_2 = \min\left\{70, [-10]_+\right\} = 0$$
$$m_3 = \min\left\{50, [-30]_+\right\} = 0, \qquad m_4 = \min\left\{40, [-50]_+\right\} = 0.$$

In order to prevent this situation (individuals with a negative income after redistribution, or individuals above their claims, when it is not possible to satisfy all claims), we present the following modification of the *additive solidarity* axiom, which introduces an equal treatment of two individuals when their *minimal rights* change equally and both individuals have a positive income after redistribution.

Axiom 5. *Additive Solidarity for equal significant changes in minimal rights* (AS^{**}). Given two economies $e = (y_N, z_N), e' = (y'_N, z_N) \in \mathcal{E}$, that only differ in skills profiles, and a claims vector $\hat{\mathbf{g}}$, such that $R' < R$, if $\Delta m_i = \Delta m_j$, then $u_j(e', \hat{\mathbf{g}}) = [u_j(e, \hat{\mathbf{g}}) + \Delta u_i]_+$.

It is important to note that whenever $u_j(e', \hat{\mathbf{g}}) > 0$, then AS^{**} coincides with AS^*. So, the modification of this axiom only has relevance for values of R sufficiently low. Specifically, when an individual already receives a zero income after redistribution, that individual should no longer be affected by a further deterioration of the skills profile in society. As mentioned in Luttens (2010) "our ethical intuition may lead us to consider a minimal amount of redistribution, that we at least want to perform. Suppose that the poorest in society could not satisfy their basic needs when they receive a negative income after redistribution. Society wants to exclude this possibility in every situation by incorporating the requirement of a non-negative income after redistribution for all individuals in the construction of the redistribution mechanism."

The following example shows the different result we obtain, by applying AS^{**} instead of AS^*, in the situation of Example 1.

Example 2. (continues from Example 1) Note that now, when applying AS^{**} ($R' = 0$, $R = 150$), an egalitarian distribution is not necessarily obtained for R, since from $\mathbf{u} = (0,0,0,0)$ not necessarily all increments must be equal in order to fulfill AS^{**}. In fact, to obtain the utility vector in $R = 150$, we can begin with $R_0 = 230$, and *claims feasibility* implies $\mathbf{u} = (80,70,50,30)$. In this case, the *minimal rights* vector is $m = (80,70,50,30)$. If, due to a change in the skills profile, we now have $R_1 = 200$, then $m = (50,40,20,0)$, so axiom AS^{**} implies an equal sharing of the lost resources, that is $\mathbf{u} = (72.5, 62.5, 42.5, 22.5)$. Now, consider that the resources are $R_2 = 180$, then $m = (30, 20, 0, 0)$, so by AS^{**} and PRI, we obtain $\mathbf{u} = (65.83, 55.83, 35.83, 22.5)$. When $R_3 = 160$, then $m = (10, 0, 0, 0)$, so AS^{**} and PRI imply $\mathbf{u} = (55.83, 45.83, 35.83, 22.5)$. Finally, for $R_4 = 150$, we know $m = (0, 0, 0, 0)$, and AS^{**} and PRI imply that $\mathbf{u} = (45.83, 45.83, 35.83, 22.5)$ which differs from the egalitarian proposal obtained under AS^{*}.

3. A respect-of-minimal-rights egalitarian mechanism

This section provides an alternative to Luttens' mechanisms, which is based on the fulfillment of the respect of minimal rights axiom. Our main result characterizes this mechanism in terms of *claims feasibility*, *priority* and our new axiom of *additive solidarity for equal significant changes in minimal rights*. It must be noticed that given an economy e and a claims vector $\hat{\mathbf{g}}$, associated to any redistribution mechanism S, the utility each individual obtains is

$$u_i(e, \hat{\mathbf{g}}) = S(e, \hat{\mathbf{g}}) + v(y_i, z_i).$$

Observe that the axioms are formulated in terms of utilities, instead of the redistribution mechanism.

Our mechanism has an egalitarian objective, which is constrained in terms of the minimal right of each individual: that is, individuals with identical minimal rights increase their utility in the same level. Figure 1 shows how slopes of such individuals coincide. Observe that when the aggregate pre-tax income R is large enough $(R \geq C_{-4})$, then all individuals increase their utility at the same rate.

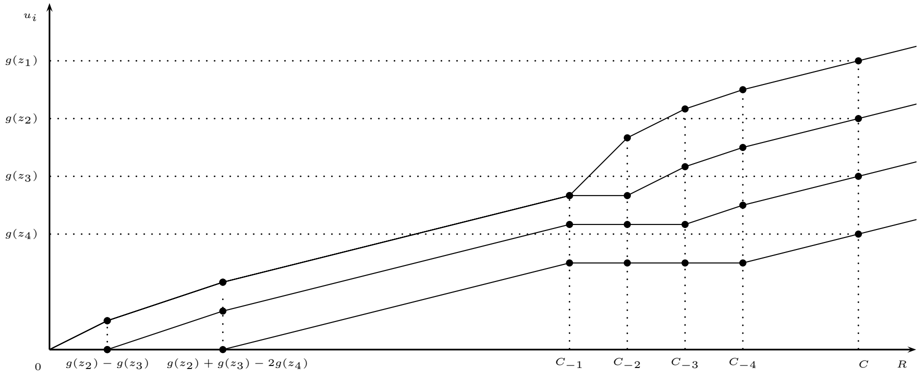

Figure 1. Utilities provided by S_{RMRE} (respect-of-minimal-rights egalitarian mechanism). The top utility level corresponds to the individual with greatest effort, u_1, then u_2, u_3 and u_4.

Definition 3. The *respect-of-minimal-rights* egalitarian mechanism S_{RMRE} allocates resources for each $e \in \mathcal{E}$, each claims vector $\hat{\mathbf{g}}$ and each $i \in N$, as follows:

$$(x_i)_{S_{RMRE}} = -v(y_i, z_i) + d_i(\hat{\mathbf{g}}, R),$$

where $d_i(\hat{\mathbf{g}}, R)$ is defined by:

(i) $R \geq C_{-n}$:

$$d_i(\hat{\mathbf{g}}, R) = g(z_i) + \frac{R - C}{n} \qquad \forall i \in N$$

(ii) $C_{-k} \leq R \leq C_{-(k+1)}$:

$$d_i(\hat{\mathbf{g}}, R) = \begin{cases} d_i(\hat{\mathbf{g}}, C_{-(k+1)}) & \forall i \geq k+1 \\ d_i(\hat{\mathbf{g}}, C_{-(k+1)}) + \dfrac{R - C_{-(k+1)}}{k} & \forall i < k+1 \end{cases}$$

(iii) $G_{n-1} \leq R \leq C_{-1}$:

$$d_i(\hat{\mathbf{g}}, R) = d_i(\hat{\mathbf{g}}, C_{-1}) + \frac{R - C_{-1}}{n} \qquad \forall i \in N$$

(iv) $G_{n+1-k} \leq R \leq G_{n+2-k}, \quad k = 2, 3, \ldots, n-2,$

$$d_i(\hat{\mathbf{g}}, R) = \begin{cases} 0 & \forall i \geq n+2-k \\ d_i(\hat{\mathbf{g}}, G_{n+2-k}) + \dfrac{R - G_{n+2-k}}{k} & \forall i < n+2-k \end{cases}$$

where $G_s = \sum_{i=2}^{s-1} g(z_i) - (s-2) g(z_{s-1})$.

Theorem 1. *A redistribution mechanism S coincides with S_{RMRE} if and only if S satisfies CF, AS** and PRI.*

Proof. See Appendix A1.

The independence of the axioms that appear in Theorem 1 is shown in Appendix A2. The following proposition, which can be straightforwardly obtained from the proof of Theorem 1, highlights the fact that our solution meets the boundedness on which it is based. On the other hand, as we have mentioned, *respect of minimal rights* implies the *participation* property.

Proposition 1. *S_{RMRE} satisfies RMR and P.*

4. Conclusions

In this paper we have analyzed redistribution problems by means of a lower bound on what individuals deserve. We have modified the mechanism proposed by Luttens (2010) so that our proposal not only makes *claims feasibility* and *participation* compatible, but also it fulfills the bound on which is based: *respect of minimal rights* (*RMR*). Our proposal behaves as the *CF*-mechanism of Luttens for a large level of resources. But we obtain that for a small level of resources, (i) no-one can incur a negative income, and (ii) no-one can receive more than their claim when the resources are not enough to satisfy the aggregate claim (*claim-boundedness*), two usual requirements in conflicting claims problems. Figure 1 shows how our mechanism works, where the horizontal and vertical axes represent different levels of the resources and the total income received by each individual, in a four-individual problem, respectively.

Finally an interesting ongoing issue is to analyze the behavior of this kind of egalitarian mechanism whenever other lower bounds considered in the literature are used.

Acknowledgement We acknowledge the comments and valuable suggestions of two anonymous referees and the Editor that have substantially improved the paper. This has been partially supported by the Spanish Ministry of Economy and Competitiveness funds under Project ECO2013-43119 and by Universitat Rovira i Virgili, Banco Santander and Generalitat de Catalunya under the project 2011LINE-06.

References

Bossert, W. (1995). Redistribution Mechanisms Based on Individual Factors. *Mathematical Social Sciences*, 29, 1–17.

Bossert, W. and Fleurbaey, M. (1996). Redistribution and Compensation. *Social Choice and Welfare*, 13, 343–355.

Fleurbaey, M. (1994). On Fair Compensation. *Theory and Decision*, 36, 277–307.

Fleurbaey, M. (2008). *Fairness, Responsibility, and Welfare*. Oxford, Oxford University Press.

Fleurbaey, M. and Maniquet, F. (2011). Compensation and Responsibility. In Arrow, A. K., and Sen, K. (eds.), *Handbook of Social Choice and Welfare*, vol. 2. Amsterdam, Elsevier/New Holland, p. 507–604.

Foley, D. K. (1967). Resource Allocation and the Public Sector. *Yale Economics Essays*, 7(1), 45–98.

Iturbe-Ormaetxe, I. (1997). Redistribution and Individual Characteristics. *Review of Economic Design*, (3), 45–55.

Kolm, S. C. (1972). *Justice et équité*. Paris, Editions du CNRS.

Luttens, R. I. (2010). Minimal Rights Based Solidarity. *Social Choice and Welfare*, 34(1), 47–64.

Maniquet, F. (1998). An Equal Right Solution to the Compensation-Responsibility Dilemma. *Mathematical Social Sciences*, 35(2), 671–687.

O'Neill, B. (1982). A Problem of Rights Arbitration from the Talmud. *Mathematical Social Sciences*, 2(4), 345–371.

Panzer, E. and Schmeidler, D. (1974). A Difficulty in the Concept of Fairness. *Review of Economic Studies*, 41(3), 441–443.

Pazner, E. and Schmeidler, D. (1978). Egalitarian Equivalent Allocations: A New Concept of Economic Equity. *Quaterly Journal of Economics*, 92, 671–687.

Rawls, J. (1971). *A Theory of Justice*. Cambridge, MA, Harvard University Press.

Thomson, W. (2003). Axiomatic and Game-Theoretic Analysis of Bankruptcy and Taxation Problems: A Survey. *Mathematical Social Sciences*, 45(3), 249–297.

Tungodden, B. (2005). Responsibility and Redistribution: The Case of First Best Taxation. *Social Choice and Welfare*, 24, 33–44.

Varian, H. (1974). Equity, Envy and Efficiency. *Journal of Economic Theory*, (9), 63–91.

Appendix

A1. Proof of Theorem 1

Given an economy $e = (y_N, z_N)$, and a claims vector $\hat{\mathbf{g}}$, we define the economy $e' = (y'_N, z_N)$ where y'_N is chosen such that $R' = C$. Since the efforts does not change, the claims vector of this new economy is $\hat{\mathbf{g}}$. Note that, for each $i \in N$, $m_i(e', \hat{\mathbf{g}}) = g(z_i)$. Hence the *CF* axiom implies that the "initial income" is $u_i(e', \hat{\mathbf{g}}) = g(z_i)$, for each $i \in N$. One of the following situations occurs:

(i) $R \geq C$

For each $i \in N$, $R - C_{-i} \geq C - C_{-i} = g(z_i)$. Thus, $m_i(e, \hat{\mathbf{g}}) = g(z_i)$, and $m_i(e, \hat{\mathbf{g}}) - m_i(e', \hat{\mathbf{g}}) = 0$. By AS^{**}, $u_i(e, \hat{\mathbf{g}}) = g(z_i) + \frac{R-C}{n}$, which coincides with (i) of Definition 3.

(ii) $C_{-n} \leq R < C$

For each $i \in N$, $R - C_{-i} = (R - C_{-n}) + (C_{-n} - C_{-i}) = (R - C_{-n}) + (g(z_i) - g(z_n)) = (R - C) + g(z_i) < g(z_i)$, and $R - C_{-i} \geq C_{-n} - C_{-i} = g(z_i) - g(z_n) \geq 0$. Thus, $m_i(e, \hat{\mathbf{g}}) = (R - C) + g(z_i)$. Hence, $m_i(e, \hat{\mathbf{g}}) - m_i(e', \hat{\mathbf{g}}) = R - R' = R - C$. By AS^{**}, $u_i(e, \hat{\mathbf{g}}) = g(z_i) + \frac{R-C}{n}$, which coincides with (i) of Definition 3.

(iii) $C_{-(n-1)} \leq R < C_{-n}$

Consider now the economy $e' = (y'_N, z_N)$ where y'_N is chosen such that $R' = C_{-n}$. Since the efforts does not change, the claims vector of this new economy is $\hat{\mathbf{g}}$. From (ii) we know that $u_i(e', \hat{\mathbf{g}}) = g(z_i) + \frac{C_{-n}-C}{n}$. For each $i \in N$, $R - C_{-i} = (R - C_{-n}) + (C_{-n} - C_{-i}) = (R - C_{-n}) + (g(z_i) - g(z_n)) = (R - C) + g(z_i) < g(z_i)$. For each $i \leq n-1$, $R - C_{-i} \geq 0$, and $R - C_{-n} < 0$. Thus, $m_i(e, \hat{\mathbf{g}}) = (R - C) + g(z_i)$ for each $i \leq n-1$ and $m_n(e, \hat{\mathbf{g}}) = 0$. Hence, $m_i(e, \hat{\mathbf{g}}) - m_i(e', \hat{\mathbf{g}}) = (R - C) + g(z_i) - (C_{-n} - C + g(z_i)) = R - C_{-n}$, for each $i \leq n-1$ and $m_n(e, \hat{\mathbf{g}}) - m_n(e', \hat{\mathbf{g}}) = 0$. By *PRI* and AS^{**}, $u_i(e, \hat{\mathbf{g}}) = u_i(e', \hat{\mathbf{g}}) + \frac{R-C_{-n}}{n-1}$, for each $i \leq n-1$ and $u_n(e, \hat{\mathbf{g}}) = u_n(e', \hat{\mathbf{g}})$ which coincides with (ii) of Definition 3.

(iv) $C_{-(k-1)} \leq R < C_{-k}$, $k = 2, 3, \ldots, n-1$

The proof of this case is completely analogous to that in (iii), just by considering the economy $e' = (y'_N, z_N)$ where y'_N is chosen such that $R' = C_{-k}$.

(v) $G_{n-1} \leq R < C_{-1}$

We consider the economy $e' = (y'_N, z_N)$ where y'_N is chosen such that $R' = C_{-1}$. Since the efforts does not change, the claims vector of this new economy is $\hat{\mathbf{g}}$. For each $i \in N$, $R - C_{-i} = (R - C_{-1}) + (C_{-1} - C_{-i}) = (R - C_{-1}) + (g(z_i) - g(z_1)) < 0$. Thus, $m_i(e, \hat{\mathbf{g}}) = 0$. Hence, $m_i(e, \hat{\mathbf{g}}) - m_i(e', \hat{\mathbf{g}}) = 0$. By AS^{**}, $u_i(e, \hat{\mathbf{g}}) = u_i(e', \hat{\mathbf{g}}) + \frac{R-C_{-1}}{n}$, which coincides with (iii) of Definition 3.

(vi) $G_{n+1-k} \leq R < G_{n+2-k}$, for $k = 2, 3, \ldots, n-2$

Finally, consider the economy $e' = (y'_N, z_N)$ where y'_N is chosen such that $R' = G_{n+2-k}$. Since the efforts does not change, the claims vector of this new economy is $\hat{\mathbf{g}}$. For each $i \in N$, $R - C_{-i} < 0$. Thus, $m_i(e,\hat{\mathbf{g}}) = 0$. Hence, $m_i(e,\hat{\mathbf{g}}) - m_i(e',\hat{\mathbf{g}}) = 0$. By AS^{**}, $u_i(e,\hat{\mathbf{g}}) = u_i(e',\hat{\mathbf{g}}) + \frac{R-G_{n+2-k}}{k}$, for each $i \leq n+2-(k+1)$ and $u_i(e,\hat{\mathbf{g}}) = 0$, otherwise, which coincides with (iv) of Definition 3.

Finally, it is immediate to observe that the utility functions obtained from the redistribution mechanism S_{RMRE} satisfy the required axioms. □

A2. Axioms Independence

In the next examples we show that the axioms in Theorem 1 are independent. In all of them, there are three individuals which are decreasingly ordered, as usually, that is, $g_1 = g(z_1) \geq g_2 = g(z_2) \geq g_3 = g(z_3)$.

(i) **A mechanism fulfilling** CF, AS^{**} **and not** PRI.

Let S_a the mechanism defined by:

(a) If $R < C$ and $m_3 < m_2 = m_1 < g_2$, then $u_i = \frac{R}{3}$.

(b) $S_a = S_{RMRE}$, otherwise.

It is clear that CF and AS^{**} are fulfilled.

Nonetheless, S_a does no satisfy PRI, as the next numerical example shows. Consider two economies $e = (y_N, z_N)$, $e' = (y'_N, z_N) \in \mathcal{E}$, such that $R = 3$, $R' = 12$, and $\hat{\mathbf{g}} = (9,9,1)$. Then, $u(e,\hat{\mathbf{g}}) = (1,1,1)$, $m(e,\hat{\mathbf{g}}) = (0,0,0)$, and $u(e',\hat{\mathbf{g}}) = (4,4,4)$, $m(e',\hat{\mathbf{g}}) = (1,1,0)$. So S_a does not satisfy PRI.

(ii) **A mechanism fulfilling** CF**,** PRI **and not** AS^{**}.

Let S_b the mechanism defined by:

$$u_1 = R - u_2 - u_3, \quad u_2 = m_2, \quad u_3 = m_3.$$

It is clear that S_b satisfies CF and PRI.

Nonetheless, S_b does no satisfy AS^{**} as the next numerical example shows. Consider two economies $e = (y_N, z_N)$, $e' = (y'_N, z_N) \in \mathcal{E}$, such that $R = 12$, $R' = 15$, and $\hat{\mathbf{g}} = (10,9,1)$. Then, $u(e,\hat{\mathbf{g}}) = (11,1,0)$, $m(e,\hat{\mathbf{g}}) = (2,1,0)$, and $u(e',\hat{\mathbf{g}}) = (11,4,0)$, $m(e',\hat{\mathbf{g}}) = (5,4,0)$. So S_b does not satisfy AS^{**}.

(iii) **A mechanism fulfilling** PRI, AS^{**} **and not** CF.

Let S_c the mechanism defined by:

(a) If $m_3 < m_2 = m_1 \leq g_2$, then $u_3 = g_3$, and $u_2 = u_1 = \frac{R-g_3}{2}$.

(b) If $m_3 < m_2 = g_2 < m_1$, then $u_3 = g_3$, $u_2 = \frac{3g_2-g_3}{2}$, and $u_1 = R - u_2 - u_3$.

(c) $S_c = S_{RMRE}$, otherwise.

It is clear that *PRI* and AS^{**} are fulfilled.

Nonetheless, S_c does no satisfy *CF* as the next numerical example shows. Consider an economy $e = (y_N, z_N) \in \mathcal{E}$, such that $R = 21$, and $\hat{\mathbf{g}} = (14, 6, 1)$. Then, $m(e, \hat{\mathbf{g}}) = (14, 6, 1)$ and $u(e, \hat{\mathbf{g}}) = (11.5, 8.5, 1)$. So S_c does not satisfy *CF*.