Czech Economic Review 6 (2012) 199–208
Acta Universitatis Carolinae Oeconomica

Dynamic Collusion and Collusion Games in Knaster's Procedure

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Received 26 May 2012; Accepted 6 September 2012

Abstract In this paper we study the collusion in Knaster's procedure, starting from the paper of Fragnelli and Marina (2009). First, we introduce a suitable dynamic mechanism, so that the coalition enlargement is always non-disadvantageous. Then, we define a new class of TU-games in order to evaluate the collusion power of the agents.

Keywords Fairness, Knaster's procedure, complete risk aversion, dynamic coalition formation
JEL classification C70, C71

1. Introduction

The collusion is a secret and fraudulent agreement among two or more agents for an illicit purpose, at damage of other ones. This general definition reflects an ancient and frequent practice which affects various sectors, from policy to markets (price cartel, cartel of banks and trust). Collusion was already known in ancient Rome, as attested by the term *collusio* used by Cicero and Seneca. Auctions and sports are also not immune from collusion. For example, auctions with low minimum prices are vulnerable to collusion among bidders. Graham and Marshall (1987) study the optimal minimum price set by a seller, while Mead (1967) and Milgrom (1987) prove that ascending-bid auction is more susceptible to collusion than sealed-bid auction. To avoid that judges of artistic sports collude, Federations adopt various strategies in the regulations (Gambarelli et al. 2012). We recall that the collusive behavior is illegal. For instance, the Italian Civil Code punishes the suspected or supposed colluders.

In this paper, starting from the results in Fragnelli and Marina (2009), we analyze the problem of collusion in Knaster's procedure (1946), a tool which allocates indivisible objects with monetary compensations in order to restore fairness: each indivisible item is first exchanged for money equal to the highest valuation of it, then the monetary quantity is shared among all the agents according to their valuations. Knaster's procedure is *efficient* (there is no other distribution that yields every agent

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a higher payoff), and *proportional* (each of the n agents thinks to receive at least one n-th of the total value), if the agents report their true valuations (Brams and Taylor 1996, 1999); but it is subject to the manipulation. When an agent misreports her/his valuation individually, Knaster's procedure is manipulable, since there is no Pareto efficient and *strategy-proof* (non-manipulable) procedure (Holmström 1979), where strategy-proofness means that truthful report is a dominant strategy (Svensson 1999, 2009). Using a different concept of manipulation, Knaster's procedure with infinitely risk-averse agents is non-manipulable, since there is no way of obtaining a *safe* gain (Fragnelli and Marina 2009). If two or more agents (but not all) coordinate their false declarations, Knaster's procedure proves to be coalition-manipulable, where a mechanism is said *coalition-strategy-proof* when "*if a joint misreport by a coalition strictly benefits one member of the coalition, it must strictly hurt at least one other member*" (Moulin 1993). We follow the line started in the paper by Fragnelli and Marina (2009), making the same assumptions and adopting the same definitions. They show that the gain produced by the agreement is always non-negative, even if enlarging the set of colluders the gain may increase or decrease.

The aim of this paper is twofold: to analyze whether the collusion from a dynamic viewpoint, i.e. after enlarging the set of colluders, may be always profitable, despite the non-monotonicity of the gain; then we propose a way to measure the relevance of each agent in the collusion mechanism. More precisely, first we prove that, through a suitable dynamic mechanism, the coalition formation process is always non-disadvantageous, since the previous gain of the incumbent colluders is guaranteed by their altered declarations, so they secure themselves against the entrant colluders; second, we introduce the class of collusion games and use the Shapley value to evaluate the influence of each agent.

The paper is organized as follows. Game theoretic definitions and notions we refer to in the paper are presented in Section 2. In Section 3 we recall Knaster's procedure applied to a single indivisible object and describe the collusion mechanism for Knaster's procedure among completely risk-averse agents. In Section 4 the dynamic process of coalition formation is illustrated. In Section 5 we propose two possible division schemes. Section 6 introduces the TU-game associated to collusion, focusing on the Shapley value. Section 7 concludes.

2. Recalls of game theory

In this section, we shortly present some classical concepts in game theory.

A *Transferable Utility game* or *TU-game* in characteristic function form is a pair $\langle N, v\rangle$, where $N = \{1,\ldots,n\}$ is a finite *set of players* and $v : 2^N \to \mathbb{R}$ is a real function, with $v(\emptyset) = 0$, called *characteristic function*. A subset $S \subset N$ is called *coalition* and N is called *grand coalition*. $v(S), S \subseteq N$, is the *worth* of S, i.e. the utility that the players in S may obtain independently from the other players. We say that a TU-game $\langle N, v\rangle$ is *inessential*, if $v(N) = \sum_{i\in N} v(\{i\})$; *monotonic*, if $v(S) \le v(T)$ whenever $S \subseteq T$; *cohesive*, if $v(N) \ge \sum_{j=1,..,k} v(S_j)$ for any partition $\{S_1,\ldots,S_k\}$ of N; *superadditive*, if $v(S\cup T) \ge v(S)+v(T)$ whenever $S\cap T = \emptyset$; *convex*, if $v(S\cup T)+v(S\cap T) \ge v(S)+v(T)$

whenever $S,T \subseteq N$. Given a TU-game $\langle N,v\rangle$, an *imputation* is a vector $(x_i)_{i\in N} \in \mathbb{R}^n$, such that $\sum_{i\in N} x_i = v(N)$ (*efficiency*) and $x_i \geq v(\{i\})$ for each $i \in N$ (*individual rationality*). The *core* of a game is the set of imputations such that $\sum_{i\in S} x_i \geq v(S)$ for each $S \subset N$ (*coalitional rationality*).

An *allocation rule* for a TU-game is a function ψ which assigns an allocation $\psi(v)$ to every TU-game $\langle N,v\rangle$ in the class of games with player set N. One of the most usual rules is the *Shapley value* (Shapley 1953), φ, given by $\varphi_i(v) = \sum_{S\subseteq N: i\notin S} \left[\big((n-s)!(s-1)!\big)/n!\right] \cdot (v(S\cup\{i\}) - v(S))$, for all $i \in N$, where s is the cardinality of S.

3. Preliminaries

In this section we recall the concepts of Knaster's procedure (1946) and collusion among infinitely risk-averse agents (Fragnelli and Marina, 2009).

3.1 Knaster's procedure for one object

Applying the Knaster's procedure, we suppose that the value of each object obtained by an agent is independent from who has obtained the other objects (*additivity*), so the problem of allocating a set of objects simply corresponds to treating each object independently, as Fink wrote to Brams (mentioned in Brams and Taylor 1996).

Let $N = \{1,\dots,n\}$ be the set of agents, which we assume to be completely risk-averse, to have the same valuation of monetary quantities, and to have equal rights on the object. We suppose that agent $i \in N$ knows only her/his own *valuation* v_i of the item and does not use any statistical information on the valuations of the others. We assume also that agents are not subject to any liquidity or budget constraints. Without loss of generality, we assume that the agents are ordered according to weakly decreasing valuations, i.e. $v_1 \geq v_2 \geq \ldots \geq v_n$ and, in case of multiple maximal valuations, we call 1 the agent who receives the object at the end of the procedure. After that the valuations are communicated to a mediator, agent 1, that has the highest valuation v_1, gets the object and pays her/his valuation; exchanging the indivisible item for money makes the division possible. Each agent $i \in N$ receives the expected *initial fair share* $E_i = (1/n)\cdot v_i$, plus an equal share of the *surplus* $s = v_1 - (1/n)\cdot\sum_{j\in N} v_j$. The surplus is non-negative (Brams and Taylor 1996), in particular it is zero if and only if all the valuations are exactly the same (Kuhn 1967). The *adjusted fair share* or payoff of agent $i \in N$ is $V_i = E_i + s/n = v_i/n + v_1/n - (1/n^2)\cdot\sum_{j\in N} v_j$.

At the end of the allocation, the sum of the compensations in money $c_1 = (1-n/n)\cdot v_1 + s/n$ and $c_i = v_i/n + s/n$, for $i \in N\setminus\{1\}$ is zero, so Knaster's procedure does not require or produce money (Brams and Taylor 1996). The solution is proportional since it secures to agent $i \in N$ a portion $V_i \geq (1/n)\cdot v_i$. Knaster's procedure with more than two agents does not guarantee envy-freeness, as an agent may prefer another's portion to her/his own. For instance, agent k may envy agent j, with $1 < j < k \leq n$, whenever $c_j > c_k$, i.e. $v_j > v_k$.

3.2 Collusion

Some agents may decide to coordinate their declarations and misreport their valuations in order to increase their shares in Knaster's procedure. According to Fragnelli and Marina (2009), the colluders, after exchanging information on their valuations, agree both on altering their declarations and on sharing the safe gain.

Definition 1. *A collusion of a coalition of completely risk-averse agents consists of three elements: a revelation of their true valuations, a binding agreement on the same declaration of their highest true valuation and a binding agreement on the gain sharing.*

We remark that a binding agreement is necessary, since at least the agent with the highest declaration suffers a loss, but s/he accepts to collude if the other partners assure for him a compensation. Moreover, we can notice that the colluding groups are disjoint, as an agent colluding with two groups is unmasked when the bids are made.

Formally, let $C = \{i_1, \dots, i_c\} \subset N$ be the set of colluders with agents ordered according to weakly decreasing valuations, i.e. $v_{i_1} \geq v_{i_2} \geq \dots \geq v_{i_c}$. It is sufficient to study *partial* collusion (agreement among a part of the agents), since the gain of *total* collusion (agreement among all the agents) is always zero. According to Definition 1, all the colluders agree on declaring $b^C = v_{i_1}$. At the end of the procedure, the colluders with a gain compensate the colluders with a loss and then they share the total gain $G^C = \left((n-c)/n^2\right) \cdot \sum_{k \in C}(b^C - v_k)$. Each agent outside the colluding coalition suffers the same loss as the others, i.e. $G^C/(n-c) = \left(1/n^2\right) \cdot \sum_{k \in C}(b^C - v_k)$.

The collusion gain is always non-negative, in particular it is null only if the collusion is total or if the colluders have the same valuation. The profit from collusion is non-monotonic with respect to the number of colluders: when c increases there is one more non-negative term in the summation, but $n - c$ decreases.

To make the previous concepts clearer, we revise Example 1 in Fragnelli and Marina (2009).

Example 1. The true valuations of four completely risk-averse agents, 1, 2, 3, and 4, for an indivisible object and their payoffs are as indicated in Table 1.

Table 1. Initial situation: no collusion

Agent	1	2	3	4
True valuation	240	192	80	48
Payoff	85	73	45	37

Assume (Scenario 1) that agents 1 and 4 collude declaring 240 units (see Table 2; colluders declarations are underlined). Agent 4 refunds with her/his gain of 36 units the loss of agent 1 of 12 units, then they share the safe gain $G^{\{1,4\}} = 24$.

Now, assume (Scenario 2) that the collusion involves agents 1 and 2 (see Table 3). Referring to the initial situation, agent 2 refunds with her/his gain of 9 units the loss of

Table 2. Scenario 1: collusion between agents 1 and 4

Agent	1	2	3	4
Declaration	240	192	80	240
Payoff	73	61	33	73

Table 3. Scenario 2: collusion between agents 1 and 2

Agent	1	2	3	4
Declaration	240	240	80	48
Payoff	82	82	42	34

agent 1 of 3 units, then they share the safe gain $G^{\{1,2\}} = 6$.

Finally, assume (Scenario 3) that the collusion involves agents 1, 2, and 4 (see Table 4). Referring to the initial situation, agents 1 and 2 have a loss of 15 and 3 units, respectively, while agent 4 has a gain of 33 units, so s/he compensates the partners and then the three agents share the safe gain $G^{\{1,2,4\}} = 15$.

Table 4. Scenario 3: collusion between agents 1, 2, and 4

Agent	1	2	3	4
Declaration	240	240	80	240
Payoff	70	70	30	70

Example 1 shows that the gain has no monotonicity property, as $G^{\{1,2\}} < G^{\{1,2,4\}} < G^{\{1,4\}}$, and also that $G^{\{1,2\}}/2 < G^{\{1,2,4\}}/3 < G^{\{1,4\}}/2$. If we analyze the collusion as a *static* mechanism, i.e. supposing that the colluding coalition is formed after a unique negotiation, we may conclude that the colluders cannot *a priori* be in favor or against a larger group.

Since the gain G^C depends on n, c, the valuations of the colluders, and is independent from the declarations of the other agents, each colluder knows the true valuations of the others and, then, the gain, only during the first phase (truthful revelation of the valuations) of formation of the collusion. In other words, first s/he chooses the partners, then s/he knows the gain. On the other hand, if we analyze the collusion as a *dynamic* mechanism, that is dependently on how the coalition is formed, we can obtain a weakly profitable process for enlarging the colluder set, since who enters the collusion may compensate the incumbents for the loss caused by her/his entrance. It is sufficient that the incumbent colluders communicate to the entrant colluders the same valuations, according to their previous colluding agreement.

4. Coalition formation: subsidizations and compensations

In this section we prove that the dynamic mechanism of coalition formation gives advantageous results despite the potential reduction of total gain. Let $\mathcal{I} = \{i_1, \ldots, i_i\} \subset N$ be the set of *incumbents*, ordered according to weakly decreasing valuations, who had already colluded, agreeing on declaring $b^{\mathcal{I}} = v_{i_1}$ and on the gain sharing; let $\mathcal{E} = \{j_1, \ldots, j_e\} \subseteq N \setminus \mathcal{I}$ be the set of *entrants*, ordered according to weakly decreasing valuations. The agents in the new colluding coalition $C = \mathcal{I} \cup \mathcal{E}$ (note that $|C| = c = i + e$) agree on declaring $b^C = \max(b^{\mathcal{I}}, b^{\mathcal{E}})$, where $b^{\mathcal{E}} = v_{j_1}$, and on a gain sharing.

As it is said in Fragnelli and Marina (2009), "*the gain of the colluders is independent from any fixed (false or true) declaration of the agents not involved in the collusion. Consequently, if two or more groups of colluders form, the variation of the total payoff of each group is not affected by the formation of the others. Of course the final payoff of each agent depends on the declaration of all the agents*" (Fragnelli and Marina 2009, p. 150), then it is not important whether the agents in $\mathcal{E}$ have colluded among them or not and whether the set $\mathcal{I}$ has been formed in a sequence of steps or only in one step. Consequently, it is sufficient to analyze only one stage of the coalition formation process.

The dynamic mechanism is non-disadvantageous if the situation of the agents in C after the enlargement is weakly preferable to the previous situation of the agents in $\mathcal{I}$ and in $\mathcal{E}$, i.e. if $G^C \geq G^{\mathcal{I}} - L^{\mathcal{I}}(\mathcal{E})$, where $L^{\mathcal{I}}(\mathcal{E})$ denotes the total loss of the agents in $\mathcal{E}$ after the collusion of the agents in $\mathcal{I}$. Note that

$$G^C = \frac{n-c}{n^2} \sum_{k \in C} (b^C - v_k) = \frac{n-c}{n^2} \left(\sum_{k \in \mathcal{I}} (b^C - b^{\mathcal{I}}) + \sum_{k \in \mathcal{I}} (b^{\mathcal{I}} - v_k) + \sum_{k \in \mathcal{E}} (b^C - v_k) \right)$$

and

$$G^{\mathcal{I}} - L^{\mathcal{I}}(\mathcal{E}) = \frac{n-i}{n^2} \sum_{k \in \mathcal{I}} (b^{\mathcal{I}} - v_k) - \frac{e}{n^2} \sum_{k \in \mathcal{I}} (b^{\mathcal{I}} - v_k) = \frac{n-c}{n^2} \sum_{k \in \mathcal{I}} (b^{\mathcal{I}} - v_k),$$

so

$$G^C - \left(G^{\mathcal{I}} - L^{\mathcal{I}}(\mathcal{E})\right) = \frac{n-c}{n^2} \left(\sum_{k \in \mathcal{I}} (b^C - b^{\mathcal{I}}) + \sum_{k \in \mathcal{E}} (b^C - v_k) \right) \geq 0.$$

This means that $G^C + L^{\mathcal{I}}(\mathcal{E}) \geq G^{\mathcal{I}}$, i.e. the entrants use their initial loss $L^{\mathcal{I}}(\mathcal{E})$ to subsidize the possible reduction of gain corresponding to $G^C - G^{\mathcal{I}}$ of the incumbents due to the enlargement; then the non-negative gain $G^{\mathcal{I},\mathcal{E}} = G^C + L^{\mathcal{I}}(\mathcal{E}) - G^{\mathcal{I}}$ is shared among the agents in C. $G^{\mathcal{I},\mathcal{E}}$ may be viewed as the gain of the enlargement of the colluding group from $\mathcal{I}$ to $\mathcal{I} \cup \mathcal{E}$ after that the agents in $\mathcal{E}$ refund the (possible) loss of the agents in $\mathcal{I}$.

We can apply the dynamic mechanism to the situation in Example 1.

Example 2. Suppose that agents 1 and 4 collude, with a gain of 24 units and that agent 2 enters the collusion reducing the total gain, in the static model, to 15. In the dynamic model, we may provide a profitable situation to the three agents. Comparing

Scenarios 1 and 3, agent 2 has a reduction of her/his loss of 9 units that is used to refund the loss of 3 units of both agents 1 and 4, generating the gain $G^{\{1,4\},\{2\}} = 3$.

Analogously, suppose that agents 1 and 2 collude, with a gain of 6 units and that agent 4 enters the collusion. Comparing Scenarios 2 and 3, agent 4 has a gain of 36 units that is used to refund the loss of 12 units of both agents 1 and 2, generating the gain $G^{\{1,2\},\{4\}} = 12$.

Finally, suppose (Scenario 4) that also agent 3 enters the collusion with the other three agents (see Table 5). Comparing Scenarios 3 and 4, agent 3 has a reduction of her/his loss of 30 units that is used to refund the loss of 10 units of the agents 1, 2 and 4, generating the gain $G^{\{1,2,4\},\{3\}} = 0$, so the colluders become aware of being the grand coalition.

Table 5. Scenario 4: collusion between all the agents

Agent	1	2	3	4
Declaration	$\underline{240}$	$\underline{240}$	$\underline{240}$	$\underline{240}$
Payoff	60	60	60	60

5. Allocation of the gain

In this section, we propose two rules for allocating the gain of the collusion in the dynamic mechanism, the *equal division rule* (EQ) and the *division proportional to the original valuations* (PROP).

(i) *Equal division rule*.

At each step of the collusion, the extra gain $G^{\mathcal{I},\mathcal{E}}$ is equally shared among all the agents. So, the agents in $\mathcal{I}$ obtain $G^{\mathcal{I},\mathcal{E}}/c + G^{\mathcal{I}}/i$ and the agents in $\mathcal{E}$ obtain $G^{\mathcal{E},\mathcal{I}}/c$.

(ii) *Division proportional to original declaration rule*.

At each step of the collusion, the extra gain $G^{\mathcal{I},\mathcal{E}}$ is shared among all the agents proportionally to their declaration at the step when the collusion started. This means that each agent $i \in \mathcal{I}$ refer to her/his false declaration $b^{\mathcal{I}}$ while each agent $j \in \mathcal{E}$ refer to her/his valuation v_j.

In the following example we apply these two rules to the situation in Scenario 3 of Example 1.

Example 3. Suppose that agent 2 enters collusion $\{1,4\}$. With the EQ-rule the gain for agents 1 and 4 is $\frac{3}{3} + \frac{24}{2}$, so their final payoffs are 98 and 50 units, respectively; for agent 2 the gain is $\frac{3}{3}$ and her/his final payoff is 62.

With the PROP-rule the gains for agents 1 and 4 are $\frac{240}{240+240+192} \times 3 + \frac{240}{240+48} \times 24 = 21.07$ and $\frac{240}{240+240+192} \times 3 + \frac{48}{240+48} \times 24 = 5.07$, respectively, so their final payoffs are 106.07 and 42.07, respectively; for agent 2 the gain is $\frac{192}{240+240+192} \times 3 = 0.86$ and her/his final payoff is 61.86.

Remark 1. *The equal division coincides with the Nash solution of the bargaining problem (Nash 1950) in which the feasible set contains all the possible allocations. The equal division may assign to an agent with a very low valuation more than her/his value of the item (see agent* 4 *in Example 3), but this is not a real problem as it may happen also in the Knaster's procedure without collusion.*

6. Collusion game

In this section, we introduce a new class of games, the *collusion games* whose player set is a subset of the agents of a division situation and whose characteristic function assigns to each coalition the value of the collusion of its members in the situation at hand. More precisely, fixed a group of agents $S = \{i_1, \ldots, i_s\} \subseteq N$, the collusion game of the players in S is $\langle S, v_S \rangle$, with v_S defined by

$$v_S(C) = \frac{n-c}{n^2} \sum_{k \in C} (b^C - v_k), \quad \forall C \subseteq S.$$

Note that the gain of coalition C in the game with player set S depends on the number of agents in N. The value of $v_S(C)$ is zero when C is a singleton, so the imputation set is always non-empty, while the core may be empty or not. The core is always non-empty when $|S| = 2$ and it coincides with the imputation set, while for larger player sets nothing may be stated, as the following example shows.

Example 4. Consider a situation with $N = \{1,2,3,4,5\}$ whose true valuations are 22, 22, 22, 2 and 2, respectively. If $S = \{1,2,3\}$ we have the null game that has non-empty core, while if $S = \{2,3,4\}$ we have $v_S(23) = 0, v_S(24) = v_S(34) = 2.4, v_S(234) = 1.6$ so the core is empty.

When $S = N$ the game is inessential, but it is interesting for evaluating the relevance of the agents in the static collusion mechanism. In particular, we suggest to use the Shapley value as a measure of the collusion power of each player. As the game v_N is inessential, the Shapley value has positive and negative components, except for the null game; players with positive values may profit, on average, from a static collusion, while players with negative values suffer, on average, from a static collusion. A theoretical application is an ex-ante analysis of the profitability for an agent to participate in a colluding group, supposing we know the ex-post valuations, like an impartial external observer with complete information. The Shapley value may be viewed as an insurance against collusions, that the agents with negative values pay to the agents with positive values. Of course, from a practical point of view it is difficult to have such a complete information.

Example 5. *Consider the situation in Example 1; in this case the Shapley value when all the agents collude is* $\varphi(v_N) = \left(\frac{8}{3}, -\frac{16}{3}, -\frac{2}{3}, \frac{10}{3}\right)$.

The possibility of negative components of the Shapley value also when S is not the grand coalition makes it not suitable as an allocation rule of the gains, so the rules presented in Section 5 are more appealing, and also more easy to compute.

Example 6. Consider the situation in Example 4; if agents 1, 2, 3 and 4 collude, the Shapley value of the game $v_{\{1,2,3,4\}}$ assigns a negative amount to players 1, 2 and 3.

Finally, it is easy to verify the lack of several properties for the games $\langle S, v_S\rangle, S \subseteq N$. Referring to the situation in Example 1, let $S = \{1,3,4\}$, the game v_S, is non-monotonic, since $v_S(\{1,3,4\}) = 22 < 24 = v_S(\{1,4\})$, it has empty core, and consequently, it is neither cohesive nor convex and it is non-superadditive, since $v_S(\{1,4\}) + v_S(\{3\}) > v_S(\{1,3,4\})$.

7. Concluding remarks

In the static collusion, an external observer or the agents involved in the collusion may have interest in finding the optimal coalition. This research is open to several interpretations. One can study the maximal total utility coalitions, that is argmax $v(C)$, or the maximal *per capita* utility coalitions, that is argmax $v(C)/c$. On the other hand, in the dynamic process of coalition formation, it is difficult to answer to the question on the optimal coalition. The agents out of the colluding group are interested in entering the collusion, since they may increase their profits, or at least decrease their losses after the collusion of the incumbents. So, in the dynamic perspective, the coalition enlargement is never disadvantageous, since the incumbent colluders guarantee themselves the previous gain by their altered declarations and secure themselves against the entrant colluders. Referring to Example 1, the optimal collusion seems to be $\{1,4\}$ since $G^{\{1,4\}} = 24$, but, when agent 2 adds to the incumbent colluders they share the extra gain $G^{\{1,4\},\{2\}}$.

Acknowledgment The authors gratefully acknowledge two anonymous referees.

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