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Voting Experiments: Measuring Vulnerability of Voting Procedures to Manipulation

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Abstract A minimal reduction in strategic voter's knowledge about other voters' voting patterns severely limits her ability to strategically manipulate the voting outcome. In this paper I relax the implicit assumption made in the Gibbard-Satterthwaite's impossibility theorem about strategic voter‘s complete information about all other voters' preference profiles. Via a series of computation-based simulations I find that vulnerability to strategic voting is decreasing in the number of voters and increasing in the number of alternatives. Least vulnerable voting procedures are Condorcet-consistent procedures, followed by elimination procedures, while most prone to manipulation are the simplest rules. Strategic voting is vulnerable both to an absolute and relative reduction in amount of information.

Keywords Voting, manipulation, information, computation-based simulations
JEL classification C72, D72, D81

1. Introduction

Strategic voting disturbs the optimality of the social choice and collective decisions. Under strategic voting voters wilfully vote for alternatives, which they would otherwise abandon had they faced different respective probabilities of their success. Voting for other than voter's best (order of) alternatives may lead to implementation of publicly inferior projects or to undertaking collectively undesired investments. Due to strategic voting the ruling power may be delegated to controversial or illegitimate representatives.

Strategic voting is not only predicted by economic theory (Myerson and Weber 1993; Feldman and Serrano 2006; Edlin et al. 2007) but is also an empirically documented voting pattern (Alvarez and Nagler 2000; Blais et al. 2001; Schmitt 2001). Fisher (2001a, 2001b) claims that the sophisticated voters, who out of their short-term instrumental motivations want to best influence the election result, misrepresent in elections their individual voting preference pattern.

The motivation for this paper comes exactly from an observation that typically a non-negligible portion of the electorate votes for their second or third best alternatives. Voters do so because they realize that their most preferred alternatives face in expectation low probabilities of electoral success. At other instances, voters support

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coalitional partners to their preferred parties, so that they would strengthen the overall potential of the forming coalition. Or in multiple-round presidential elections it may pay off to support weaker undesired candidates in the earlier rounds, so that the preferred candidate wins more easily in the final round.

Despite of the evident rationality of the strategic voting at the individual level, strategic voting may be often suboptimal in terms of the social welfare. Preference misrepresentation typically leads to decisions, which are not desirable.

Nevertheless, as will be shown in this article the ability to strategically manipulate the voting outcome may hinge crucially on the amount of information that the strategic agent possesses. This article reacts on the Gibbard-Satterthwaite theorem (Gibbard 1973; Satterhwaithe 1975), which predicts susceptibility to strategic manipulation for all non-dictatorial and universal voting procedures. The impossibility result is dismal in the sense that all feasible voting procedures are vulnerable to strategic manipulation and hence it is impossible to come at a socially desirable decision through voting. The focus of the reaction centres on the implicit assumption made in the theorem that strategic voter possesses full information about all other voters‘ preferences. This information enables the agent to perform the voting manipulation successfully. In this article I am going to study the impact of relaxation of such an assumption.

Via a series of computation-based simulations of voting I aim to assess the role of strategic voter's knowledge on her ability to successfully select an individually optimal voting pattern. To do so I first estimate, which procedures are most-to-least vulnerable to strategic voting. Successively I ask how the probability to strategically manipulate behaves under constrained information of the strategic voter. The strategic voter will not know the full preference profile. This is important both for checking of the robustness of the ranking of most-to least vulnerable procedures and for the insights into restrained informational setting per se. By standard econometric analysis I analyse the systematic patterns of the probability to manipulate with regard to the number of voters and number of competing alternatives. It will lead to predictions, which sizes of committees or public boards would in general most discourage their members from strategic considerations. Last I look at the relationship between the intensity of information, which voting procedures require from voters to disclose in the election process and strategic voter's ability to successfully manipulate the procedure under the constrained information.

The main findings are that strategic voting is vulnerable both to an absolute and relative reduction in the amount of possessed information. That is a positive result with regard to the dismal predictions of Gibbard-Satterthwaithe's theorem. A minimal reduction in the agent's holding of information severely threatens her ability to strategically manipulate the voting outcome. I show that the vulnerability of voting procedures is a diminishing function of the number of participants, and an increasing function of the number of competing alternatives. Consistently, I show that strategic manipulation is most vulnerable to reduction of information especially in the least information intensive procedures.

The following methodological section conceptualizes strategy-proofness and introduces a metric for quantifying strategic manipulation. Right after, I describe the methodology of the voting simulations and I outline the strategic voter's behaviour under different informational settings. The third section presents the results of the voting simulations, at first under full informational settings, then under constrained information. The last section concludes by summarizing the main findings.

2. Methodology and theoretical framework

Let's assume n individual voters with complete, transitive and anti-symmetric preference relations R_i on the set of alternatives X, where i is an index for individual voter and separate alternatives are denoted as $[x_1, x_2, \ldots, x_m]$. Let the total number of competing alternatives be denoted by m. The assumptions on voters' preference relations are equivalent for any R_i to be characterised as a total preference ordering.

2.1 Strategy-proofness and distance function specifications

In broad sense, a voting procedure is considered as strategy-proof if a strategic voter cannot through misrepresentation of her preferences achieve a more favourable social ranking than she would have achieved if she had voted sincerely.

Previous literature distinguishes multiple approaches to formal modelling of the strategy-proofness. Umezawa (2009) provides an overview of literature, which makes explicit references to expected utilities, where probability measures over alternatives are given (Feldman 1980; Barbera et al. 2001; Rodriguez-Alvarez 2007). The other approach defines strategy-proofness in a non-probabilistic framework, where individuals evaluate the sets of alternatives by focusing on the best and/or the worst alternative in the set (Bandyopadhyay 1983; Barbera 1977; Pattanaik 1973).

A compromise between the two approaches consists in a specification of strategy-proofness based on probabilistically measured expected utilities, which are however not over specific alternatives but over voting distances between individual's honest preference ordering and the social outcome (which is also represented in a form of an ordering). Bossert and Storcken (1992) use Kemeny's distance to evaluate the voting distances between preference orderings. Duddy et al. (2009) investigate the problem of constructing a social welfare function that is non-manipulable in a context, where individuals attempt to manipulate a social ordering as opposed to a social choice. It is this midway that I take in this article, with an important distinction of using Euclidian rather than Kemeny's metric to represent the evaluation criterion of particular social orderings.

For the purpose of cardinalization of individual's utility I need to use a metric that reflects both individual's preference and the aggregated social preference ordering. For this purpose, let us consider a set of alternatives $X = (x_1, x_2, \ldots, x_m)$ and an individual strict ordering $\pi = (x_{j_1}, x_{j_2}, \ldots, x_{j_m})$. The individual ordering π can be represented by its indices $x(\pi) = (x_1(\pi), x_2(\pi), \ldots, x_m(\pi))$, where $x_k(\pi) = j_k$ (order number of alternative x_k in ordering π). Let $f(t)$ be a non-negative decreasing function of t. Then a weight $r_k(\pi)$ of alternative k in the strict ordering π can be defined as $r_k(\pi) = f(x_k(\pi))$. Distance between two strict orderings π_1 and π_2 can be measured as a distance between two vectors $r(x(\pi_1))$ and $r(x(\pi_2))$. Generally, any distance function based on vector norms would suffice, although I use Euclidian metric in my specification. Let me denote the distance between two orderings π_1 and π_2 as $d(\pi_1, \pi_2)$. Minimization of distance between individual's preference ordering and the social preference ordering corresponds to strategic voter's utility maximization.

The weights that I attach to alternatives are unified in the article. I assume them to correspond to scores, by which an individual would evaluate alternatives in *Borda's voting*. Individual's best preferred alternative is associated with a weight of $(m-1)$ points. The weights attached to consecutive options are consecutively dropping by 1 point with the last option being weighted by 0.

Consider an example of two strict orderings of 4 alternatives. Individual 1 prefers alternative a to alternative b to alternative c to alternative d. Individual 2 prefers c to d to a to b. By the considered specification and assumed weights, we can represent the two orderings by two vectors of weights $r(x(\pi_1)) = (3,2,1,0)$ and $r(x(\pi_2)) = (1,0,3,2)$. The resulting Euclidian distance between the two vectors is by a back-of-the-envelope calculation equal to $d(\pi_1, \pi_2) = 4$.

Due to the nature of the considered voting procedures, the social outcome may involve tie(s) between alternatives. This means that the alternatives may also be weakly ordered in the voting outcome. I resolve the issue by calculating an average of the distances between the individual ordering and all potential social orderings that might have arisen given random breaking of the tie(s). The list of voting procedures and their brief description is provided in Section A3 in Appendix.

2.2 Preference generating cultures

I use computation-based simulations to randomly generate a collective preference profile of a set of voters. The random generation of a collective preference profile is in the literature commonly referred to as a '*culture*'. An overview of different preference generating cultures has been provided by Laslier (2010). Particularly suitable culture for simulations in this article is the '*impartial culture*', because it is most typically used for a general evaluation of voting systems, such as where one does not want to account for any specific prior information about the alternatives nor for any assumption on the shape of distribution of voting preferences.

An essential characteristic of the impartial culture is that the preference orderings attributed to individuals are *distributed according to a uniform distribution* over $m!$ logical strict preference orderings. The impartial culture hence treats all alternatives symmetrically and learning some feature about preference orderings of some voters yields no information about the preferences of other voters.

I simulate the collective preference profile of $n-1$ voters and m alternatives 100,000 times for each considered voting procedure. I consider ten most common voting procedures, many of which are used in daily practical life. I simulated the voting process for $n = \{2,3,5,7,11\}$. I use these values since I focus on voting manipulation in small groups or committees, where the assumption that a particular voter might know all or majority of other voters' preferences is feasible.

2.3 Voting behaviour given the level of information

2.3.1 Knowledge of the full collective preference profile

The voting scenario in this baseline specification is as follows: all but one of voters cast their votes sincerely in order to come at a collective decision. The last voter attempts to manipulate the voting through her strategic vote. Her role as of a fully informed voter is straightforward. She calculates all possible distances that could occur between her own preference ordering and the final social outcome and she selects such pattern so as to minimise the voting distance (to maximise her utility). The role is simple, because strategic voter's choice finalises the aggregation of preferences. Moreover, she is the sole strategic voter. She faces no uncertainty about voting patterns of other voters. To evaluate how successful the voter was in her endeavours, I calculate in what fraction of simulations the strategic voter successfully lowered the voting distance. Under full information the number of opportunities for manipulation is equal to the number of successful manipulations.

2.3.2 Information about full rankings of a subset of voters

The manipulating ability of a voter may be hampered by a lack of knowledge about voting patterns of other voters. Let us assume away strategic voter's full knowledge of the collective preference profile. The reduction of information consists in letting the strategic voter know about the full collective preference profile except for a preference ordering of one or more of the sincere voters.

The voting scenario changes under restrained information. The strategic voter can determine only partial scores of the evaluated alternatives. Instead of the certainty, which she used to have about all of the voting profiles, she instead only expects other voters to vote according to some pre-specified probability distribution. Let the strategic voter know that the voting patterns of all unfamiliar voters are *i.i.d.* from the uniform distribution.

Now, we may think of some simple heuristic rules that the strategic voter might use given her limited information. For instance, we may think of her attempting to manipulate the partially aggregated social ordering as if it was the fully aggregated social ordering. Nevertheless such heuristic rule could often lead into situations, where the strategic voter would end up with even worse payoffs than she would achieve by voting sincerely.

Alternatively, the voter could stick to a minimalistic approach to strategic voting. She would opt for strategic voting only in cases, where the payoffs from her insincere voting strategy would never be dominated by payoffs accruing to her sincere voting.

Nonetheless, to be consistent with previous specifications, I make the voter decide for a concrete voting pattern based on a minimisation of a weighted distance between her individual ordering and all plausibly aggregated social orderings associated with that voting pattern. The weights would be the probabilities of a particular combination of voting orderings to arise.

I target to estimate the change in the success rate of strategic voter's manipulations, given the restricted information. Now, the measure of the manipulation success decomposes into two parts. First, how many times *did the strategic voter have* and how many times *did she successfully use* the opportunity to manipulate the voting outcome. The success rate may be calculated either as a number of cases when the strategic voter succeeded to lower the relevant distance. Or alternatively we may calculate the success rate as a number of cases when the voter succeeded to manipulate the voting result so as to make it copy her own individual preference ordering. I shall evaluate the former statistic.

Moreover I can evaluate such statistics as how many times the agent manipulated with *adverse consequences* to her, in the sense that the resulting social ordering was further from her own preference ordering than it would be under sincere voting. Or how do our answers change, if the strategic agent *knows of even fewer* voters' profiles? I contrast all the answers to figures obtained in cases when the agent had full information so as to obtain relative measures of successful manipulation. I consider the questions for all 10 studied voting procedures.

3. Results

3.1 Full knowledge of the collective preference profile

Table A2 in Appendix provides the complete tabulated overview of the opportunities for strategic manipulation of a sole strategic voter under full information. As I have already suggested, the number of opportunities for manipulation under full information mirrors the number of actual successful manipulations. Fully informed strategic voter cannot end up with worse payoff by voting strategically than by voting sincerely. Table 1 and Table 2 present the summary statistics on the probability of manipulation under full information by number of players and number of competing alternatives. Figure 1 and Figure 2 graphically outline the evolution of room for strategic manipulation for all considered procedures. I observe three results:

(i) Strategic manipulation opportunity levels vary substantially across the used voting procedures.
(ii) Strategic manipulation opportunity levels for 4 alternatives surpass those of 3 alternatives in every simulated procedure for all numbers of voters.
(iii) The number of sincere voters does not affect the manipulation opportunities, if we allow for wider confidence intervals, the opportunity for strategic manipulation is diminishing in the number of voters.

Table 1. Summary statistics for probability of manipulation, full information, m=3

	$n=2$	$n=3$	$n=5$	$n=7$	$n=11$
Average	0.142	0.144	0.156	0.127	0.129
Min	0	0	0	0	0
Max	0.333	0.361	0.432	0.325	0.289
Variance	0.019	0.016	0.019	0.010	0.010

Table 2. Summary statistics for probability of manipulation, full information, m=4

	$n=2$	$n=3$	$n=5$	$n=7$	$n=11$
Average	0.404	0.361	0.358	0.365	0.346
Min	0.167	0.147	0.124	0.107	0.082
Max	0.794	0.684	0.704	0.592	0.562
Variance	0.052	0.032	0.036	0.032	0.031

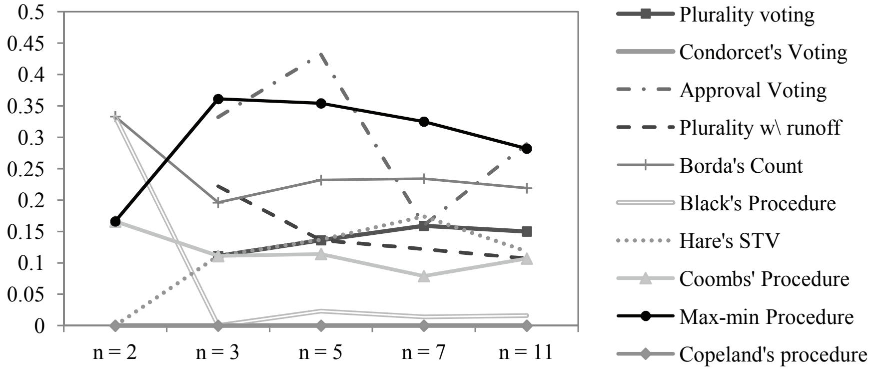

Figure 1. Probabilities of manipulation by number of players and voting procedure, full information, three alternatives

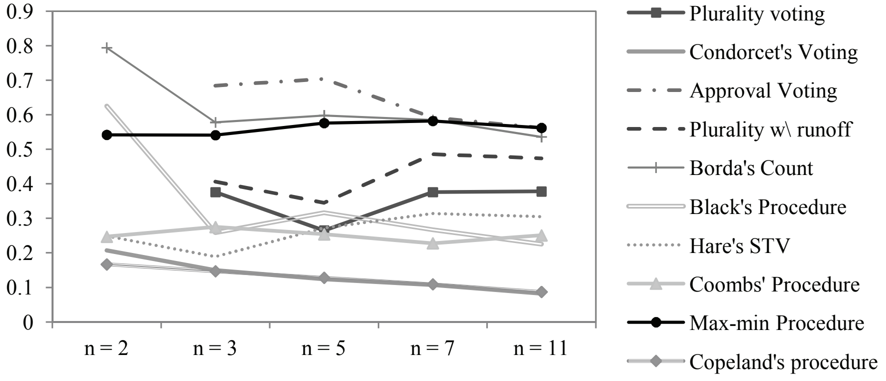

Figure 2. Probabilities of manipulation by number of players and voting procedure, full information, four alternatives

(i) Levels of strategic voting vary substantially across the used voting procedures.

Consider Figure 1 and Figure 2 in this regard. In both figures we can discern a distinct arrangement of layers. The lowest probability of manipulation can be attributed to Copeland's, Condorcet's and Black's voting procedures. This comes at no surprise, as these procedures are exactly the Condorcet-consistent procedures, in other words they always select the Condorcet's winner if it exists. The second layer of manipulability of voting procedures involves three elimination procedures: Coombs', Hare's and Plurality with runoff voting procedures. Although these procedures are not Condorcet-consistent, the probability of manipulation is only slightly higher than in the preceding group. The reason is the difficult process of consecutive rounds of eliminations, where it is not only necessary for the strategic voter to find a situation where her vote is pivotal, moreover she has to find voting pattern, which does not harm her in later rounds of eliminations.

The last most manipulable layer groups together the remaining procedures: approval voting, max-min voting and Borda's count. The common feature of these three procedures is that they allow the strategic voter to allocate wide ranges of scores to individual alternatives. This property gives to the strategic voter power to swing with scores more flexibly.

Noteworthy, some level of susceptibility to manipulation needs to be attributed also to the use of the impartial preference generating culture. Had I been using some other culture, where fewer ties would occur during the preference aggregation, the strategic voter would face fewer opportunities for gainful manipulation. That applies most apparently for procedures, where the range of points that determine the social ordering is the narrowest, e.g. max-min procedure.

In the first column of Table A1 I use simple ordinary least squares (OLS) regression to explain the variability in the susceptibility to strategic manipulation. The susceptibility is captured in the explained variable "$prob_i$". Regarding the explanatory variables n_i captures the number of players, while $m4_i$ is a dummy signifying that we choose from 4 voting alternatives rather than from 3 alternatives. The rest of the variables *Plurality* to *Copeland* are dummy variables corresponding to the 10 voting aggregation rules. They are included in the (10x1) vector "$proced_i$", to which correspond 10 coefficients contained in the (10x1) vector δ.

The formal model can be expressed by (1):

$$prob_i = \beta n_i + \gamma m4_i + \delta' proced_i + \varepsilon_i. \tag{1}$$

Index i does not stand here for an individual voter, but for a particular observation of the susceptibility to strategic manipulation. The regression coefficients allow us to rank particular procedures according to their susceptibility to manipulation consistently with previous discussion.

(ii) Susceptibility to manipulation in cases with 4 alternatives surpasses that of cases with 3 alternatives in every procedure for all considered numbers of voters.

A coefficient on number of alternatives in the first regression reads that if we are choosing from among 4 alternatives rather than from 3, there is a 25% higher chance that the strategic voter comes to a situation where it is beneficial for her to manipulate her vote. Nonetheless, it is rather sound not to generalise this result with respect to the higher numbers of competing alternatives. The pattern does not have to be increasing in the number of alternatives in the least. A sound expectation for this pattern would be to be non-linear and rather depend on the difference $(n-m)$ if not on a ratio of the number of voters and number of competing alternatives (n/m).

(iii) The number of sincere voters does not affect the manipulation opportunities under full information.

From the same regression we can observe a very slight decline of susceptibility to voting manipulation in the number of voters. We cannot reject the H_0 hypothesis of no impact of this variable at 5% confidence level, and we have to allow for wider confidence intervals to be able to reject the H_0. The logic of our expectations for the coefficient to be negative is nevertheless straightforward: the more voters are involved in a voting situation, the lesser relative weight of one vote should become, in the sense that the strategic voter becomes less often pivotal. If we evaluate the "n" coefficient for different voting procedures separately, we could conclude that it is only in Condorcet's, Copeland's and Plurality voting with runoff voting procedures with 3 alternatives that the susceptibility to manipulation decreases monotonically in the number of voters.

3.2 Incomplete information about rankings of a subset of voters

First, I have simulated a probability that the strategic voter *attempts for strategic manipulation*. The strategic voter does not have the full information, so she is coerced to decide on the basis of the weighted distances whether to attempt for a manipulation or not. The number of attempts may therefore be both higher and lower than the number of cases when the strategic voting was actually optimal. The uncertainty of the voter whether to manipulate or not propagates the other three kinds of results.

Secondly, I provide the number of cases, when the strategic voter decides to manipulate and then acquires the same voting distance as she would acquire had she had the full information. This is captured in the variable of '*Maintained best manipulation*'. The variable does *not* include the cases when it was optimal for the strategic voter to vote sincerely and she correctly chose to do so. As a consequence the variable 'Maintained best manipulation' can be only equal or lower than the number of successful manipulations in the settings with full information.

Thirdly, an alternative measure of successful manipulation was produced. It is a probability that the strategic voter on the grounds of a weighted distance chose such voting pattern, which yielded not necessarily the best voting distance, but nonetheless *better* distance *than sincere voting* would yield. Table A3 provides the results. I do not

Table 3. Summary statistics for measures of individual manipulation success

	Obs.	Mean	Std. Dev.	Min	Max
Attempts	72	0.261	0.220	0	0.749
Maintained	72	0.114	0.099	0	0.359
Better	72	0.129	0.119	0	0.496
Worse	72	0.044	0.057	0	0.336

Table 4. Correlation table for measures of individual manipulation success

	Prob	Attempts	Worse	Better	Maintained
Prob	1				
Attempts	0.7899	1			
Worse	0.4820	0.6781	1		
Better	0.8219	0.9348	0.5989	1	
Maintained	0.7501	0.8903	0.6259	0.9627	1

intend to comment on the statistics of 'Better than sincere', whereas here the results are tightly correlated with those of 'Maintained best manipulation.'

Last, I provide a variable of the number of cases, when the attempt for voting manipulation has lead to a *worse* voting distance *than sincere* voting would lead to. Even this variable can be considered as an alternative measure of successful manipulation. The residual number of cases, i.e. (100,000 simulations—'Worse than sincere') captures the number of cases when the strategic voter either correctly decided to manipulate or decided incorrectly but the voting distance was not worse than if she had voted sincerely, or the voter decided correctly not to manipulate.

Table 3 displays the summary statistics on the listed four measures of individual manipulation success. Table 4 measures correlations between these variables and the probability of manipulation under full information. All observations on manipulability for $n = 2$ were dropped together with the observations of non-manipulable Condorcet's and Copeland's procedures for $m = 3$.

3.2.1 Maintained best manipulation

The results, provided in Table A4 can be summarised in five points:

(i) the levels of susceptibility to manipulation of various procedures differ less significantly under reduced information than under full information;

(ii) we observe a rapid drop in the susceptibility to manipulation of all considered procedures under limited information, i.e. we confirm the severe vulnerability of strategic voting to an absolute reduction in information owned;

(iii) the order of the most susceptible to the least susceptible voting procedure remains unchanged when compared to the order of procedures under full information;
(iv) the susceptibility to strategic manipulation grows in the number of voters under reduced information, i.e. we confirm the vulnerability of strategic voting to a relative reduction in possessed information;
(v) the levels of manipulability are higher for cases with more alternatives.

(i) The lower variability in the susceptibility across different voting procedures is well observable from both Figures 3 and 4, and can be also documented on a lower dispersion in coefficients on Procedures in OLS regression in the second column of Table A1. Numerous coefficients are found not significantly different from zero.

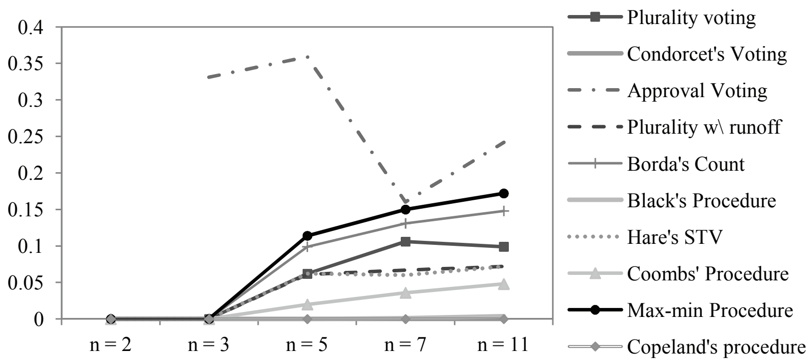

Figure 3. Probabilities of manipulation by number of players and voting procedure, reduced information, three alternatives

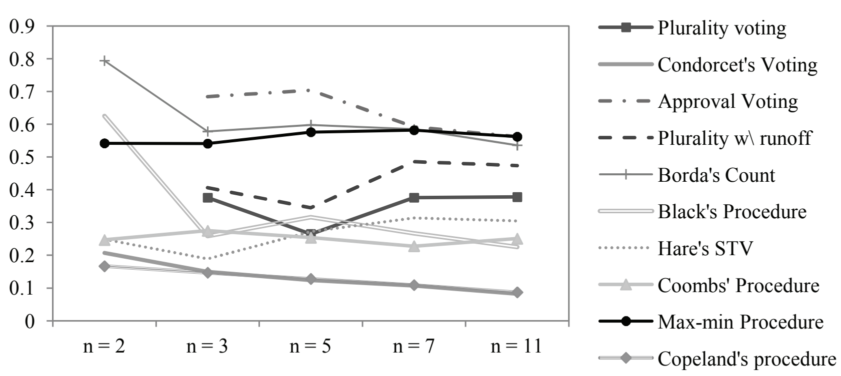

Figure 4. Probabilities of manipulation by number of players and voting procedure, reduced information, four alternatives

(ii) The rapid drop in the susceptibility to voting manipulation is best observable from the regression in third column in Table A1. Here the formal model resembles the previous model, apart from the facts that the explained variable prob_i includes both probabilities from full and reduced informational settings, and an additional explanatory dummy variable $(\text{Reduced info})_i$ controls for this difference.

A formal model follows in (2):

$$prob_i = \beta n_i + \gamma m4_i + \delta' proced_i + \lambda (Reducedinfo)_i + \varepsilon_i \tag{2}$$

The regression results suggest that the reduction in the knowledge of strategic voter about a preference profile of one sincere voter reduces the probability of maintaining the best voting manipulation by 18%. This is moreover only a partial reduction as we need to take into regard also a decline in coefficients of particular voting procedures. Lower amount of information depresses all these coefficients simultaneously.

Speaking in absolute terms, under limited information none of the voting procedures is susceptible to manipulation in more than 35% of cases. Under 3 alternatives, the level of 35% is only approached by the approval voting procedure. Disregarding approval voting, the level of susceptibility would not overcome 18%.

(iii) The order of manipulability of individual voting procedures stayed unchanged. We again find the Condorcet-consistent procedures to be least manipulable, followed by the elimination-based procedures, placing the approval, Borda's and max-min procedures at the highest ranks in the order of the least to the most manipulable voting procedures.

Next, I order the procedures according to their ***vulnerability to information reduction***, i.e. according to their ***information intensity***. I construct a ratio of 'Maintained best manipulation' over 'Probability of successful manipulation' from under full information. The lower is the ratio, the more vulnerable to information reduction the procedure is. I present the ratios in Table 5.

Table 5. Vulnerability of voting procedures to reduction in information

Procedure (a)	Obs.	Mean	Min	Max
Plurality	8	0.471	0	0.66
Condorcet	4	0.149	0	0.30
Approval	8	0.705	0.36	1
Runoff	8	0.426	0	0.67
Borda	8	0.316	0	0.67
Black	7	0.219	0	0.31
Hare's STV	8	0.463	0	0.67
Coombs	8	0.307	0	0.50
Max-min	8	0.384	0	0.60
Copeland	4	0.173	0	0.35

(a) Maintained best manipulation/probability of successful manipulation under full information

The least vulnerable procedures are the approval, plurality, Hare's and plurality with runoff procedures. The most vulnerable are Condorcet's, Copeland's and Black's procedures.

We observe that the order of manipulability of voting procedures is co-determined by the information intensity of the procedures. The least manipulable procedures are in the largest extent vulnerable to the reduction in the amount of information and the most susceptible procedures to manipulation do not suffer from information reduction that much.

(iv) Under reduced information the susceptibility to manipulation grows in the number of voters. The reason for the increasing manipulation in the number of voters can be attributed to the relatively lower share of withheld information from the strategic voters at higher numbers of voters. Knowing less of 1 sincere voter's profile when there are 11 voters is less important for the agent's ability of strategic manipulation than knowing less of 1 sincere voter's profile when there are just 3 voters. Hereby I confirm the vulnerability of strategic manipulation not only to an absolute reduction in the individual information, but also to a relative reduction.

(v) The levels of manipulability are higher for cases with more alternatives. The discussion is analogous to the one in the section of full information

3.2.2 Attempts for voting manipulation

Numbers of 'Attempts for voting manipulation' are provided in Table A5. Table A1 in its fourth column explains the number of attempts for manipulation by an OLS regression.

The reader must not draw direct inference from absolute figures in Section A2, since these figures ignore the correlation between the number of attempts and the actual probability of manipulation. It is natural to expect that the weighted distances bid the strategic voter to attempt for strategic manipulation more often in those procedures, which are more susceptible to manipulation. On the other hand, regressing the number of attempts on the probability of voting manipulation would induce endogeneity issues, since both variables are caused by third factors, such as by the number of voters, by the relative amount of withheld information, etc.

The formal model used hence puts on the left side of the regression the ratio of the 'number of attempts' over the 'probability of strategic manipulation under full information'. This ratio is captured in the variable $(\textit{Rel. Attempts})_i$.

The formal model (3) follows:

$$(\textit{Rel. Attempts})_i = \frac{attempts_i}{prob_i} = \beta n_i + \gamma m4_i + \delta' proced_i + \varepsilon_i. \tag{3}$$

From the regression we can say that there are only few voting procedures where the relative number of attempts significantly differs from other procedures. In other words, the strategic agent attempts relatively for strategic manipulation in majority of procedures to a comparable extent. Majority of coefficients accruing to individual voting procedures fall into the 95% confidence intervals of the coefficients of other voting procedures. Only Black's and Borda's procedures differ from the other procedures in this respect, and their relative number of attempts for manipulation is higher. Nevertheless, we will see that in the case of Black's procedure this increased number of attempts leads eventually to an increased number of adverse outcomes.

As a positive result I view the independence of the number of attempts on the number of alternatives. The agent opts for attempts for manipulation irrespectively of the number of alternatives, which makes her decision making consistent.

Thirdly, a decrease in the relative number of attempts in the number of voters can be interpreted as getting more exact in attempting for manipulation, which I perceive just as well positively.

Overall, we can see that the number of attempts exceeds the number of cases when voting manipulation was optimal by twofold or even more. Luckily for the strategic voter, in cases when she attempts for a voting manipulation and she does not succeed she brings about either a result that is equally good as sincere voting or is better than sincere voting although it is not the best manipulating option.

3.2.3 Adverse outcomes of attempting for manipulation

The results for outcomes, which are 'Worse than sincere' voting are provided in Table A6. Table A1 in its fifth column explains the results by an OLS regression.

Absolutely speaking, the voting outcome is worse than sincere voting on average in 5% of simulated situations when we are voting over 3 alternatives or in 15% of situations when we are voting over 4 alternatives. This percentage appears as a relatively small price to be paid for attempting for manipulation, given how many times the strategic agent succeeded in misrepresentation of her preferences. Moreover, since the strategic agent decides on a basis of a weighted distance, this distance is most probably not that much worse than the distance associated with sincere voting.

Speaking of relative figures, I relate the number of 'Worse than sincere outcomes' to the number of actual cases when voting manipulation was optimal. I capture the ratio of these two variables in a variable $(Rel.\ Worse)_i$. The formal model (4) follows:

$$(Rel.Worse)_i = \frac{worse_i}{prob_i} = \beta n_i + \gamma m4_i + \delta' proced_i + \varepsilon_i. \tag{4}$$

We can see that the voting procedures are not statistically distinguishable between each other in the regard of how many 'Relative worse than sincere' outcomes they deliver. The strategic agent selects on average the unsatisfactory voting pattern in a similar extent across all voting procedures.

The number of relatively worse outcomes is diminishing in the number of voters. A careful reader has noticed that to an increased number of voters we have previously attributed an increasing exactness of attempting for strategic manipulation. Now we discover that the increase in exactness extends also on the ability of attempting for such voting patterns, which do not harm the individual strategic voter relatively to her sincere voting. This increase may originate in the lowest relative share of withheld information at higher numbers of voters. The selection of unsatisfactory voting patterns is higher when selecting from 4 competing alternatives.

4. Conclusions

This study has computationally simulated 10 most common voting procedures so as to study the susceptibility of these procedures to strategic voting. This was followed by a study of vulnerability of strategic voting to variation in the amount of information that an individual strategic agent possessed.

The paper points out that even if the theory predicts that all feasible voting rules are vulnerable to strategic manipulation, in practical circumstances when the voters know little about other voters' voting patterns, the fears of the voting outcome being diverged far from social optimum might be unjustified.

In my paper I have shown that the susceptibility to voting manipulation is a decreasing function in the number of participants and an increasing function of the number of alternatives. All procedures could be characterised by their own specific extent to which they were susceptible to manipulation. The procedure-specific degree of susceptibility to manipulation was in turn dependent on the amount of information that the procedure typically required from a participating agent to disclose, in combination with the strictness of the voting procedure, which is the amount of points that the procedure allows the agent to manipulate with.

Least susceptible voting procedures were the Condorcet-consistent procedures: Black's, Copeland's and Condorcet's procedure itself. The second group of relatively more susceptible procedures involved three elimination procedures: Coombs', Hare's and Plurality with runoff. Most manipulable procedures were the plurality procedure, approval procedure, max-min voting and Borda's count.

Once I have stripped the agent from the full knowledge of the complete preference profile, I have confirmed the vulnerability of strategic voting to both an absolute and relative reduction in the amount of information. I found that strategic voting was most vulnerable to the reduction of information in the least information intensive procedures. Strategic voting was least vulnerable to a reduction of possessed information in approval, plurality, Hare's and plurality with runoff procedures and most vulnerable in Condorcet's, Copeland's and Black's procedures. The precision of selection of the best manipulating pattern was decreasing in the relative amount of information withheld from the strategic agent. Consistently, the agent has more often ended up with worse payoff than sincere voting would yield when a relatively larger share of information was withheld from her.

The article contributes to the discussion on the role of expectations formation in voting and on the importance of communication rules between public board members and public project evaluation committees. The variables through which the potential institutional optimisation of voting schemes could be performed are the size of committees, employed voting rule, information that is available to voters and the ways how voters form expectations about each others' voting patterns.

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Appendix

A1. Regression results

Table A1. Probability of successful manipulation

	Prob full info	Prob reduced info	Prob merged groups	Attempts reduced info	Worse outcome reduced info
n	-0.00567^{*}	0.00773^{***}	0.00310	-0.0589^{**}	-0.0179^{***}
	[0.00285]	[0.00183]	[0.00234]	[0.0229]	[0.00432]
$m=4$	0.250^{***}	0.0855^{***}	0.162^{***}	-0.00896	0.0846^{***}
	[0.0190]	[0.0115]	[0.0154]	[0.139]	[0.0263]
Reduced info			-0.186^{***}		
			[0.0146]		
Plurality	0.156^{***}	0.0278	0.174^{***}	2.336^{***}	0.349^{***}
	[0.0357]	[0.0210]	[0.0296]	[0.270]	[0.0510]
Condorcet	-0.0845^{*}	-0.121^{***}	-0.0212	3.042^{***}	0.106
	[0.0443]	[0.0283]	[0.0396]	[0.379]	[0.0716]
Approval	0.381^{***}	0.197^{***}	0.371^{***}	2.487^{***}	0.219^{***}
	[0.0357]	[0.0210]	[0.0296]	[0.250]	[0.0472]
Runoff	0.199^{***}	0.0264	0.195^{***}	2.118^{***}	0.225^{***}
	[0.0357]	[0.0210]	[0.0296]	[0.270]	[0.0510]
Borda	0.337^{***}	0.0160	0.253^{***}	3.697^{***}	0.210^{***}
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Black	0.114^{***}	-0.0519^{**}	0.115^{***}	6.264^{***}	0.227^{***}
	[0.0319]	[0.0210]	[0.0267]	[0.291]	[0.0550]
Hare STV	0.0935^{***}	0.00850	0.129^{***}	2.207^{***}	0.198^{***}
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Coombs	0.0898^{***}	-0.0372^{*}	0.108^{***}	2.752^{***}	0.261^{***}
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Max-min	0.336^{***}	0.0809^{***}	0.279^{***}	2.776^{***}	0.200^{***}
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Copeland	-0.0909^{**}	-0.118^{***}	-0.000472	2.940^{***}	0.189^{***}
	[0.0443]	[0.0283]	[0.0399]	[0.361]	[0.0683]
N	84	72	164	63	63

Note: Standard errors in brackets; *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

A2. Simulation tables

Table A2. Optimal number of voting manipulations, full information

Probability of manipulation		$n=2$	$n=3$	$n=5$	$n=7$	$n=11$
Plurality voting	$m=3$	*	0.111	0.136	0.159	0.150
	$m=4$	*	0.376	0.265	0.376	0.378
Condorcet's voting	$m=3$	0	0	0	0	0
	$m=4$	0.207	0.150	0.124	0.107	0.082
Approval voting	$m=3$	*	0.332	0.432	0.160	0.289
	$m=4$	*	0.684	0.704	0.592	0.562
Plurality with runoff	$m=3$	*	0.222	0.136	0.122	0.107
	$m=4$	*	0.406	0.345	0.486	0.474
Borda's count	$m=3$	0.333	0.196	0.232	0.234	0.219
	$m=4$	0.794	0.578	0.598	0.585	0.536
Black's procedure	$m=3$	0.332	0	0.023	0.014	0.016
	$m=4$	0.625	0.259	0.316	0.267	0.225
Hare's STV	$m=3$	0	0.111	0.137	0.174	0.118
	$m=4$	0.249	0.189	0.272	0.314	0.305
Coombs' procedure	$m=3$	0.166	0.111	0.114	0.079	0.107
	$m=4$	0.247	0.275	0.254	0.228	0.251
Max-min procedure	$m=3$	0.166	0.361	0.354	0.325	0.282
	$m=4$	0.542	0.541	0.576	0.582	0.562
Copelands procedure	$m=3$	0	0	0	0	0
	$m=4$	0.167	0.147	0.128	0.109	0.087

* For Plurality, Condorcet's and Approval voting procedures, the results are trivial for $n=2$.

Table A3. Alternative measure of manipulation, reduced information

Better than sincere		$n=2$	$n=3$	$n=5$	$n=7$	$n=11$
Plurality voting	$m=3$	*	0	0.062	0.106	0.099
	$m=4$	*	0.251	0.117	0.156	0.175
Condorcet's voting	$m=3$	0	0	0	0	0
	$m=4$	0	0	0.012	0.021	0.026
Approval voting	$m=3$	*	0.331	0.359	0.160	0.242
	$m=4$	*	0.496	0.315	0.439	0.407
Plurality with runoff	$m=3$	*	0	0.061	0.067	0.072
	$m=4$	*	0.171	0.130	0.239	0.247
Borda's count	$m=3$	0	0	0.099	0.131	0.148
	$m=4$	0	0.040	0.175	0.211	0.238
Black's procedure	$m=3$	0	0	0	0.001	0.004
	$m=4$	0	0.091	0.144	0.121	0.099
Hare's STV	$m=3$	0	0	0.062	0.060	0.072
	$m=4$	0	0.126	0.103	0.187	0.208
Coombs' procedure	$m=3$	0	0	0.020	0.036	0.048
	$m=4$	0	0.046	0.073	0.105	0.135
Max-min procedure	$m=3$	0	0	0.114	0.150	0.172
	$m=4$	0	0.235	0.314	0.337	0.353
Copelands procedure	$m=3$	0	0	0	0	0
	$m=4$	0	0	0.015	0.025	0.032

* For Plurality, Condorcet's and Approval voting procedures, the results are trivial for $n=2$.

Table A4. Probability to maintain optimal outcome, reduced information

Maintained		$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 11$
Plurality voting	$m = 3$	*	0	0.062	0.106	0.099
	$m = 4$	*	0.251	0.117	0.156	0.175
Condorcet's voting	$m = 3$	0	0	0	0	0
	$m = 4$	0	0	0.012	0.021	0.025
Approval voting	$m = 3$	*	0.331	0.359	0.160	0.242
	$m = 4$	*	0.253	0.264	0.352	0.357
Plurality with runoff	$m = 3$	*	0	0.061	0.067	0.072
	$m = 4$	*	0.171	0.127	0.222	0.235
Borda's count	$m = 3$	0	0	0.099	0.131	0.148
	$m = 4$	0	0.029	0.124	0.156	0.185
Black's procedure	$m = 3$	0	0	0	0.001	0.004
	$m = 4$	0	0.068	0.100	0.085	0.071
Hare' s STV	$m = 3$	0	0	0.062	0.060	0.072
	$m = 4$	0	0.126	0.100	0.187	0.205
Coombs' procedure	$m = 3$	0	0	0.020	0.036	0.048
	$m = 4$	0	0.046	0.072	0.098	0.126
Maxmin procedure	$m = 3$	0	0	0.114	0.150	0.172
	$m = 4$	0	0.132	0.241	0.278	0.304
Copelands procedure	$m = 3$	0	0	0	0	0
	$m = 4$	0	0	0.014	0.025	0.031

* For Plurality, Condorcet's and Approval voting procedures, the results are trivial for $n = 2$.

Table A5. Number of attempts for manipulation, reduced information

Attempts for manipulation		$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 11$
Plurality voting	$m = 3$	*	0	0.149	0.209	0.205
	$m = 4$	*	0.498	0.203	0.251	0.290
Condorcet's voting	$m = 3$	0	0	0	0	0
	$m = 4$	0	0	0.032	0.052	0.065
Approval voting	$m = 3$	*	0.667	0.592	0.374	0.419
	$m = 4$	*	0.749	0.542	0.729	0.707
Plurality with runoff	$m = 3$	*	0	0.109	0.132	0.146
	$m = 4$	*	0.248	0.185	0.338	0.397
Borda's count	$m = 3$	0	0	0.291	0.375	0.422
	$m = 4$	0	0.120	0.429	0.527	0.616
Black's procedure	$m = 3$	0	0	0	0.007	0.019
	$m = 4$	0	0.449	0.530	0.470	0.410
Hare' s STV	$m = 3$	0	0	0.111	0.134	0.151
	$m = 4$	0	0.257	0.130	0.265	0.338
Coombs' procedure	$m = 3$	0	0	0.057	0.083	0.110
	$m = 4$	0	0.088	0.194	0.234	0.239
Max-min procedure	$m = 3$	0	0	0.184	0.277	0.347
	$m = 4$	0	0.505	0.627	0.666	0.671
Copelands procedure	$m = 3$	0	0	0	0	0
	$m = 4$	0	0	0.049	0.066	0.080

* For Plurality, Condorcet's and Approval voting procedures, the results are trivial for $n = 2$.

Table A6. Worse outcomes than sincere voting would yield, reduced information

Worse than sincere		$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 11$
Plurality voting	$m = 3$	*	0	0.037	0.040	0.022
	$m = 4$	*	0.247	0.058	0.085	0.069
Condorcet's voting	$m = 3$	0	0	0	0	0
	$m = 4$	0	0	0.0006	0.001	0.008
Approval voting	$m = 3$	*	0.336	0.111	0	0.047
	$m = 4$	*	0.061	0.050	0.066	0.067
Plurality with runoff	$m = 3$	*	0	0	0.007	0.002
	$m = 4$	*	0.077	0.043	0.079	0.076
Borda's count	$m = 3$	0	0	0.041	0.042	0.033
	$m = 4$	0	0.027	0.063	0.072	0.058
Black's procedure	$m = 3$	0	0	0	0	0.001
	$m = 4$	0	0.156	0.108	0.080	0.057
Hare's STV	$m = 3$	0	0	0	0	0
	$m = 4$	0	0.130	0.017	0.034	0.028
Coombs' procedure	$m = 3$	0	0	0.018	0.002	0.016
	$m = 4$	0	0.021	0.052	0.030	0.041
Max-min procedure	$m = 3$	0	0	0.009	0.011	0.011
	$m = 4$	0	0.133	0.122	0.108	0.082
Copelands procedure	$m = 3$	0	0	0	0	0
	$m = 4$	0	0	0.011	0.010	0.009

* For Plurality, Condorcet's and Approval voting procedures, the results are trivial for $n = 2$.

A3. List of simulated voting procedures

For details on these procedures see Nurmi (1987).

(i) **Simple plurality voting**

Each voter needs to decide, to which single alternative to assign a score of 1, while assigning 0 to all other alternatives. Plurality winner is such an alternative that collects the highest number of votes.

(ii) **Condorcet's voting procedure**

A winning alternative is chosen by Condorcet's procedure if and only if the alternative is not defeated by a strict majority by any other alternative in a pair-wise vote.

(iii) **Plurality voting with runoff**

Plurality voting with runoff involves two rounds. The first round proceeds just like simple plurality voting. The second round involves a vote between two alternatives with the highest scores obtained in the first round. The purpose of the first round, so-called runoff is to eliminate the least preferred options.

(iv) **Borda's voting procedure**

Given m alternatives each voter's first ranked alternative obtains (m-1) points, second ranked alternative obtains (m-2) points, the third one gets (m-3) points, and so forth, down to a minimum of 0 points for the last alternative. The scores are added up and the option with the highest score becomes the **Borda's winner.**

(v) **Approval voting**

Individual voter may assign a score of 1 to as many alternatives as she wishes and assign nothing to all other alternatives. An alternative with highest score wins.

(vi) **Black's voting procedure**

Black's procedure simply chooses the Condorcet's winner if one exists. Otherwise it chooses the winner and ranks the alternatives according to Borda's voting.

(vii) **Hare's single transferable vote system**

If some alternative in Hare's voting procedure is ranked first by more than 50% of voters, it wins the election. If none such alternative exists, the alternative with fewest first ranks is eliminated from the count and the rest of alternatives is being pushed upwards in the preference lists of the voters. We again determine if any alternative ranks first by more than 50% of the voters. If so, it becomes a winner. If not, another round of eliminations proceeds. Eventually, after a number of rounds of eliminations one alternative must become Hare's winner or a tie is established in the final round.

(viii) **Coombs' technique**

Coombs suggested a slight modification to Hare's voting procedure and that was to eliminate during the rounds of elimination such an alternative that is ranked last by the largest number of voters. The qualification criterion for victory stayed the absolute majority of the first ranks in voters' preference profiles.

(ix) **Max-min voting technique**

Max-min procedure counts how many voters rank an alternative above each of other alternatives. For every alternative the procedure finds the lowest of these numbers. The procedure then ranks the alternatives according to the retrieved minima.

(x) **Copeland's voting procedure**

The procedure attributes a number of wins and a number of losses to each alternative. The alternative wins over other alternative, if it gains a majority of votes in a pair-wise vote. Otherwise it loses. The social ordering consists of an ordered list of differences between a sum of wins and sum of losses of each alternative. Copeland's procedure obviously selects the Condorcet's winner if it exists.