**AUCO Czech Economic Review 5 (2011) 162–171**
Acta Universitatis Carolinae Oeconomica

# Equilibrium Solution in a Game between a Cooperative and its Members

**Cesarino Bertini**[*], **Gianfranco Gambarelli**[**], **Antonino Scarelli**[†], **Zoltán Varga**[‡]

Received 24 May 2010; Accepted 30 April 2011

**Abstract** In the paper a game-theoretical model is set up to describe the conflict situation in which the members of a marketing cooperative may take advantage of an external market price, higher than that offered by the cooperative. Under appropriate conditions on the penalty strategy of the cooperative, the faithfulness of all members will provide a Nash equilibrium for the considered game, which at the same time also is an attractive solution, with the cooperative as a distinguished player.

**Keywords** Cooperatives, Game Theory, Nash equilibria
**JEL classification** C02, C70, Q13

## 1. Introduction

An agricultural cooperative in a given region may perform several activities, ranging from product processing to complex marketing, see e.g. Cobia (1989). Producers of a given product often form a marketing cooperative, only for the commercialization of their product. A typical situation is when a marketing cooperative negotiates a contracted price with large buyers, sharing risk among members of the cooperative. However, by the time of the actual commercialization of the product, the direct market price may be higher than that the cooperative can guarantee for members, negotiated on beforehand. Some "unfaithful" members may be interested in selling at least a part of their product outside the cooperative. The latter however, can punish them for it. This conflict situation is described in terms of a game-theoretical model.

This one-step conflict situation was studied in Larbani and Scarelli (2004). A dynamic evolutionary game approach was applied in Varga et al. (2010), also dealing with the case of oligopoly market conditions.

[*] University of Bergamo, Department of Mathematics, Statistics, Informatics and Applications, Bergamo, Italy. E-mail: cesarino.bertini@unibg.it.
[**] Corresponding author. University of Bergamo, Department of Mathematics, Statistics, Computer Science and Applications, via dei Caniana 2, 24127 Bergamo, Italy. E-mail: gambarex@unibg.it, Phone: +39 035 205 2515.
[†] University of Tuscia, Department of Ecology and Sustainable Economic Development, Viterbo, Italy. E-mail: scarelli@unitus.it.
[‡] Szent István University, Institute of Mathematics and Informatics, Gödöllő, Hungary. E-mail: Varga.Zoltan@gek.szie.hu.

In the present paper we consider an agricultural cooperative where producers, as part of their strategy in dealing with the cooperative, at the beginning of the yearly production cycle declare a projected quantity to be sold to the cooperative at a contracted price. At the time of the harvest, as a second strategy choice, each producer decides how much of his production he would actually sell to the cooperative. If a producer does not deliver the contracted amount, he will be punished by the cooperative, proportionally to the undelivered amount and to the extra profit gained from selling on the direct market. It is up to the cooperative to set the penalty rates. If by the time of the harvest, the direct market price turns out to be lower than the contracted price guaranteed by the cooperative, the latter would not buy more than the contracted amount. We will find a Nash equilibrium solution for this conflict situation, supposing that the higher direct market price will occur with a given probability.

## 2. Model description

Let us suppose that there are $n$ members in a marketing cooperative, and the general potential production capacity of the $j$-th member is expressed by a reference value $q_j$, in practice this can be the amount sold by the member at his entrance to the cooperative, or a weighted mean of yearly productions, discounting the early years with respect to the recent ones. Hence the total production potential of the cooperative is $\sum_{j=1}^{n} q_j$. Considering a given year, for simplicity we suppose that $q_j$ is a good estimation of the production of the year, and both the contracted (promised) amount and the amount actually sold to the cooperative will be expressed as a multiple of this reference value.

Assume that at the beginning of the vegetation (or production) period, the cooperative, based on its production potential, can negotiate with a large buyer, agreeing on a unit price $Q^c$ at which the cooperative can contract the amount offered by each member. Suppose that the $j$-th member decides to contract the amount $k_j q_j$, this is his first strategic decision. By the time of harvesting, the unit price $Q^d$ of the direct market may be higher or lower than the contracted unit price $Q^c$ the cooperative offered at the beginning. We will consider two different situations and the combination of them.

**Case 1.** We suppose that the decision of the members on how much product to deliver to the cooperative, is made when the direct market price is already known to be higher than the contracted price: $Q^d > Q^c$. Then the $j$-th member may decide to sell only a part of his production, $h_j q_j$, to the cooperative $(0 \leq h_j \leq k_j)$. The cooperative, however, proportionally penalizes the unfaithful member in two ways, both for failed delivery and for the extra profit due to unfaithfulness, with respective penalty rates $p_0$ and $p_1$. The pair $(p_0, p_1)$ will be then considered the strategy choice of the cooperative. In this situation, the $j$-th member takes his second strategic decision, by the choice of $h_j$. This conflict situation can be modelled in terms of a many-player game.

For $j \in \{1, \ldots, n\}$, let the $j$-th player be the $j$-th member of the cooperative, and denote by $k_j^0 \in [0,1]$ the minimal reasonable declaration acceptable to the cooperative from member $j$, considered the $j$-th player. Then, in terms of the above notations, the strategy set of the latter, is defined as

$$A_j := \{(k_j, h_j) \in [k_j^0, 1] \times [0,1] | h_j \leq k_j\}. \tag{1}$$

With some $P_0 \geq 1$, let $p_0 \in [0, P_0]$ be the penalty rate, the cooperative sets for non-delivery of the contracted amount. Similarly, with given

$$P_1 > \frac{Q^d}{Q^d - Q^c},$$

let $p_1 \in [0, P_1]$ be the penalty rate, the cooperative sets for the extra profit a member gained from selling the non-delivered amount on the direct market. Set

$$A_{n+1} := [0, P_0] \times [0, P_1] \tag{2}$$

is considered as the strategy set of the cooperative as $(n+1)$-th player. Given a multi-strategy $((k_1, h_1), \ldots, (k_n, h_n), (p_0, p_1)) \in \times_{j=1}^{n+1} A_j$, the payoff of the $j$-th player is defined as the total gain from the cooperative and the direct market, minus the penalty paid for unfaithfulness, i.e. for having failed to deliver the contracted amount and for having sold it on the direct market:

$$\begin{aligned} F_j((k_1, h_1), \ldots, (k_n, h_n), (p_0, p_1)) := \quad & Q^d (1 - h_j) q_j + Q^c h_j q_j - p_0 Q^c (k_j - h_j) q_j \\ & - p_1 (Q^d - Q^c)(1 - h_j) q_j, \quad j \in \{1, \ldots, n\} \end{aligned} \tag{3}$$

while the payoff of the cooperative is the gain from selling the contracted and actually delivered product, minus the damage caused by non-delivery, plus the penalty collected from the unfaithful members:

$$\begin{aligned} F_{n+1}((k_1, h_1), \ldots, (k_n, h_n), (p_0, p_1)) \quad := \quad & Q^c \sum_{j=1}^{n} h_j q_j - Q^c \sum_{j=1}^{n} (k_j - h_j) q_j + \\ & + p_0 Q^c \sum_{j=1}^{n} (k_j - h_j) q_j + \\ & + p_1 (Q^d - Q^c) \sum_{j=1}^{n} (1 - h_j) q_j \end{aligned} \tag{4}$$

**Case 2.** Assume that the direct market price is less than or equal to the contracted price, $\hat{Q}^d \leq Q^c$, and the cooperative accepts only the contracted amount of the production of each member. Now it is reasonable for the members to sell the promised amount to the cooperative, and the rest on the direct market, which means $h_j = k_j$. Hence, for the respective payoff functions, for each multi-strategy $((k_1, h_1), \ldots, (k_n, h_n), (p_0, p_1)) \in \times_{j=1}^{n+1} A_j$, we have

$$\hat{F}_j((k_1, h_1), \ldots, (k_n, h_n), (p_0, p_1)) = \hat{Q}^d (1 - h_j) q_j + Q^c h_j q_j, \quad j \in \{1, \ldots, n\} \tag{5}$$

for member $j$, and

$$\widehat{F}_{n+1}((k_1, h_1), \ldots, (k_n, h_n), (p_0, p_1)) = Q^c \sum_{j=1}^{n} h_j q_j \tag{6}$$

for the cooperative. In this case, there is no real conflict situation.

**Case 3.** Suppose finally that at the time of the strategy choice, the players judge that, by the time of the commercialization $Q^d > Q^c$ will take place with probability $r \in [0,1]$, and $\hat{Q}^d \leq Q^c$ with probability $1-r$. Then, for the corresponding game the expected payoffs are

$$\Phi_j := rF_j + (1-r)\hat{F}_j, \quad j \in \{1,\ldots,n\}, \tag{7}$$

$$\Phi_{n+1} := rF_{n+1} + (1-r)\hat{F}_{n+1}. \tag{8}$$

## 3. Equilibrium solutions of the games

In this section, under the condition of a sufficiently high penalty rate for unfaithfulness, we prove the existence of equilibrium solutions (Nash equilibria) for Cases 2 and 3. An equilibrium solution means a multi-strategy in which all players are interested in the following sense: No player can be better off by deviating from his equilibrium strategy, provided the rest of the players stick to their equilibrium strategy.

**Theorem 1.** *Considering Case 1 ($Q^d > Q^c$), let us suppose the cooperative sets penalty rates $p_0^* \in [0,P_0]$, $p_1^* \in [0,P_1]$, with*

$$p_1^* \geq \frac{Q^d}{Q^d - Q^c} \tag{9}$$

*Then multistrategy $((1,1),\ldots,(1,1),(p_0^*,p_1^*))$ is a Nash equilibrium for the game with strategy sets* (1)–(2) *and payoffs* (3)–(4).

**Proof.** Fix a $j \in \{1,\ldots,n\}$, and let $(k_j,h_j) \in A_j$ with $(k_j,h_j) \neq (1,1)$. Then condition (9) obviously implies

$$Q^d(1-h_j)q_j \leq p_1(Q^d - Q^c)(1-h_j)q_j \leq p_0Q^c(k_j - h_j)q + p_1(Q^d - Q^c)(1-h_j)q_j.$$

Hence we obtain

$$Q^d(1-h_j)q_j - p_0Q^c(k_j-h_j)q - p_1(Q^d-Q^c)(1-h_j)q_j \leq 0,$$

implying the Nash inequality for the $j$-th player:

$$\begin{aligned} F_j((1,1),\ldots,(k_j,h_j),\ldots,(1,1),(p_0^*,p_1^*)) &= Q^d(1-h_j)q_j + Q^c h_j q_j - p_0^* Q^c(k_j-h_j) \\ &\quad -p_1^*(Q^d-Q^c)(1-h_j)q_j \\ &\leq Q^c h_j q_j \\ &\leq Q^c q_j = F_j((1,1),\ldots,(1,1),(p_0^*,p_1^*)). \end{aligned}$$

Furthermore, for the payoff of the cooperative, for all $(p_0,p_1) \in A_{n+1}$ we get

$$F_{n+1}((1,1),\ldots,(1,1),(p_0,p_1)) = Q^c \sum_{j=1}^{n} q_j = F_{n+1}((1,1),\ldots,(1,1),(p_0^*,p_1^*)).$$

□

For the more general Case 3, when $Q^d > Q^c$ will take place with probability $r \in [0,1]$ known to the players at the time of their strategy choice, we also have the existence of similar Nash equilibria:

**Remark 1.** *Condition (9), written in the form $p_1^*(Q^d - Q^c) \geq Q^d$, can be interpreted as follows: for the equilibrium solution the penalty rate for profit per unit of goods, obtained from selling on the direct market, must be higher than (or equal to) the unit price on the direct market.*

**Theorem 2.** *Considering Case 3, let us suppose the cooperative chooses strategy $(\bar{p}_0, \bar{p}_1) \in A_{n+1}$ with expected penalty rate for the extra profit of an unfaithful member, high enough:*

$$r\bar{p}_1 \geq \frac{Q^d}{Q^d - Q^c}. \tag{10}$$

*Then multi-strategy $((1,1),\ldots,(1,1),(\bar{p}_0,\bar{p}_1))$ is a Nash equilibrium for the game* (1), (2), (7), (8).

**Proof.** Fix a $j \in \{1,\ldots,n\}$, and let $(k_j,h_j) \in A_j$ with$(k_j,h_j) \neq (1,1)$. Now inequality (10) implies

$$Q^d \leq r\bar{p}_1(Q^d - Q^c),$$

and therefore we have

$$\begin{aligned} rQ^d(1-h_j)q_j &\leq rQ^d(1-h_j)q_j \\ &\leq r\bar{p}_1(Q^d - Q^c)(1-h_j)q \\ &\leq r[\bar{p}_0 Q^c(k_j - h_j)q + \bar{p}_1(Q^d - Q^c)(1-h_j)q_j]. \end{aligned}$$

Hence, using inequality $\hat{Q}^d \leq Q^c$, we get

$$\begin{aligned} \Phi_j((1,1),\ldots,(k_j,h_j),\ldots,(1,1),(\bar{p}_0,\bar{p}_1)) &= (1-r)(\hat{Q}^d(1-h_j)q_j + Q^c h_j q_j) + rQ^c h_j q_j \\ &\quad + r[Q^d(1-h_j)q_j - \bar{p}_0 Q^c(k_j - h_j)q - \bar{p}_1(Q^d - Q^c)(1-h_j)q_j] \\ &= (1-r)\hat{Q}^d(1-h_j)q_j + Q^c h_j q_j + \\ &\quad + r[Q^d(1-h_j)q_j - \bar{p}_0 Q^c(k_j - h_j)q - \bar{p}_1(Q^d - Q^c)(1-h_j)q_j] \\ &\leq Q^c(1-h_j)q_j + Q^c h_j q_j = Q^c q_j = \Phi_j((1,1),\ldots,(1,1),\ldots,(1,1),(\bar{p}_0,\bar{p}_1)). \end{aligned}$$

The Nash inequality for the cooperative obviously holds:

$$\begin{aligned} \Phi_{n+1}((1,1),\ldots,(1,1),(p_0,p_1)) &= rQ^c \sum_{j=1}^{n} h_j q_j (1-r)Q^c \sum_{j=1}^{n} h_j q_j = Q^c \sum_{j=1}^{n} h_j q_j \\ &= \Phi_{n+1}((1,1),\ldots,(1,1),(\bar{p}_0,\bar{p}_1)). \end{aligned}$$

□

**Remark 2.** *Similarly to Case 1, the interpretation of condition* (8), $r\bar{p}_1(Q^d - Q^c) \geq Q^d$*, can be interpreted as follows: For the equilibrium solution the expected penalty rate for the profit per unit of good, obtained from selling on the direct market, must be higher than (or equal to) the unit price on the direct market.*

**Remark 3.** *Note that, if $Q^d > Q^c$ would hold with probability $r = 1$, from Theorem 2, as a particular case Theorem 1 is obtained.*

**Remark 4.** *Case 3 also includes a more general stochastic approach in the following sense: Given a distribution of the market price, parameter r of Case 3 (i.e. the probability of $Q^d > Q^c$) can be calculated and the expected payoffs can be obtained using formulae* (7)–(8).

**Remark 5.** *It is easy to see that the Nash equilibria of Theorems 1 and 2 are also attractive solutions of the respective games, in the sense of the following definition, (see e.g. Larbani 1997 and Varga et al. 2010). Consider an N-player game where*

$X_i$ *is the strategy set of the i-th player,*

$X := \prod_{i=1}^{N} X_i$ *the set of multi-strategies,*

$F_i : X \to \mathbf{R}$ *the payoff of the i-th player,* $F := (F_1, \ldots, F_N)$.

**Definition 1.** *A multi-strategy $x^0$ is said to be an attractive solution of the normal form game (X,F), if there exists distinguished player $i \in \{1,\ldots,N\}$ such that the following conditions are satisfied:*

(i) $F_j(x_1,\ldots,x_i^0,\ldots,x_N) \le F_j(x^0)$, $j \in \{1,\ldots,n\} \setminus \{i\}$, $x_k \in X_k$, $k \in \{1,\ldots,N\} \setminus \{i\}$;

(ii) $F_i(x_1^0,\ldots,x_i,\ldots,x_N^0) \le F_i(x^0)$, $x_i \in X_i$.

**Remark 6.** *If in condition (i), for each $j \in \{1,\ldots,n\} \setminus \{i\}$ we choose $x_k := x_k^0 \in X_k$ for $k \in \{1,\ldots,N\} \setminus \{i,j\}$, and an arbitrary $x_j \in X_j$, we obtain that any attractive solution is a Nash equilibrium, too.*

The interpretation of the distinguished player is the following: if player *i* sticks to his equilibrium strategy, the rest of the players can not increase their payoff even if they deviate together from their equilibrium strategies. In the context of our game, the distinguished player is the cooperative.

## 4. Example: Marketing cooperative for the commercialization of apples

For an illustration suppose that there are six members in a marketing cooperative commercializing the apples they produce. Under the general conditions of the model description of Section 2, assume their respective production potentials $q_j$ (in quintals) are (60, 90, 60, 100, 180, 176), with a total production potential of the cooperative $\sum_{j=1}^{n} q_j = 666$.

Let $Q^c = 30$ and $Q^d = 36$ be the unit price (in euros) paid by the cooperative and the direct market, respectively. Then the threshold (8) for the penalty rate the cooperative sets for the extra profit a member would gain from selling the non-delivered amount on the direct market, is $Q^d/(Q^d - Q^c) = 6$. Now suppose the cooperative threatens the members with penalty rates $p_0^* = 0.1$ and $p_1^* = 6.1$. Theorem 1 then says that in case of total faithfulness, i.e. when each member sells all his product to the cooperative, we obtain a Nash equilibrium $((1,1),\ldots,(1,1),(p_0^*,p_1^*))$. Table 1 shows the equilibrium values of the respective payoffs $F_j$ and $F_{n+1}$. In particular, the sum of the payoffs of the members equals the payoff 19,980 of the cooperative.

Now we suppose member 4 deviates from its equilibrium strategy, choosing strategy pair (0.9, 0.8), but all other players stick to their equilibrium strategy, with $p_0^* =$

**Table 1.** With penalty rates $p_0^* = 0.1$ and $p_1^* = 6.1$, at Nash equilibrium, the sum of the payoffs of the members equals the payoff 19,980 of the cooperative.

| Member | $q_j$ | $k_j$ | $k_jq_j$ | $h_j$ | $h_jq_j$ | $(1-h_j)q_j$ | $(k_j-h_j)q_j$ | Direct | Coop | NonDel | Extra | $F_j$ | $F_{n+1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 60 | 1 | 60 | 1 | 60 | 0 | 0 | 0 | 1,800 | 0 | 0 | 1,800 | 19,980 |
| 2 | 90 | 1 | 90 | 1 | 90 | 0 | 0 | 0 | 2,700 | 0 | 0 | 2,700 | 2,700 |
| 3 | 60 | 1 | 60 | 1 | 60 | 0 | 0 | 0 | 1,800 | 0 | 0 | 1,800 | 1,800 |
| 4 | 100 | 1 | 100 | 1 | 100 | 0 | 0 | 0 | 3,000 | 0 | 0 | 3,000 | 3,000 |
| 5 | 180 | 1 | 180 | 1 | 180 | 0 | 0 | 0 | 5,400 | 0 | 0 | 5,400 | 5,400 |
| 6 | 176 | 1 | 176 | 1 | 176 | 0 | 0 | 0 | 5,280 | 0 | 0 | 5,280 | 5,280 |
| Totals | 666 | | 666 | | 666 | 0 | 0 | 0 | 19,980 | 0 | 0 | 19,980 | 19,980 |

0.1 and $p_1^* = 6.1$ for the cooperative. Table 2 shows that the payoff 2,358 of member 4 is less than its equilibrium value 3,000. The payoff of the cooperative also decreases from its equilibrium value 19,980 to 19,842. Of course, by setting a penalty rate $p_1^*$ higher than 6.1, the cooperative may try to compensate its loss.

**Table 2.** For member 4, the strategy choice $k_j = 0.9$ and $q_j = 0.8$, results in loss in payoff with respect to its equilibrium value.

| Member | $q_j$ | $k_j$ | $k_jq_j$ | $h_j$ | $h_jq_j$ | $(1-h_j)q_j$ | $(k_j-h_j)q_j$ | Direct | Coop | NonDel | Extra | $F_j$ | $F_{n+1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 60 | 1 | 60 | 1 | 60 | 0 | 0 | 0 | 1,800 | 0 | 0 | 1,800 | 1,800 |
| 2 | 90 | 1 | 90 | 1 | 90 | 0 | 0 | 0 | 2,700 | 0 | 0 | 2,700 | 2,700 |
| 3 | 60 | 1 | 60 | 1 | 60 | 0 | 0 | 0 | 1,800 | 0 | 0 | 1,800 | 1,800 |
| 4 | 100 | 0.9 | 90 | 0.8 | 80 | 20 | 10 | 720 | 2,400 | 30 | 732 | 2,358 | 2,862 |
| 5 | 180 | 1 | 180 | 1 | 180 | 0 | 0 | 0 | 5,400 | 0 | 0 | 5,400 | 5,400 |
| 6 | 176 | 1 | 176 | 1 | 176 | 0 | 0 | 0 | 5,280 | 0 | 0 | 5,280 | 5,280 |
| Totals | 666 | | 656 | | 646 | 20 | 10 | 720 | 19,380 | 30 | 732 | 19,338 | 19,842 |

Now we illustrate that the above Nash equilibrium is also an attractive solution of the game. On the basis of the production foreseen for the actual season, the $j-$th member, considering the possible external market price and that of the cooperative, as well as his behaviour strategy (faithfulness or unfaithfulness) declares to deliver amount $k_jq_j$ to the cooperative. Let (1, 0.9, 1, 0.9, 0.8, 0.94) be the vector of foreseen deliveries $k_j$. The total amount that should be delivered to the cooperative is $\underline{\underline{Q}}$= 600.4. This means that only the first and third members think to be faithful, the rest of them would retain a part of their production for sale in the external market. At the time of the actual harvest, the vector of effective deliveries $h_j$ turns out to be (0.8, 0.85, 1, 0.8, 0.75, 0.82), and the actual total delivery to the cooperative is $\sum_{j=1}^{n} h_jq_j$= 549.82, compared to 666 foreseen at the beginning of the season, see Table 3. For the cooperative, the amount of product missing with respect to the members' declaration is 116.18; 50.62

of which, at the time of delivery ended up on the direct market.

**Table 3.** If five out of six members deviate from their equilibrium strategies, while the cooperative maintains its strategy pair $p_0^* = 0.1$ and $p_1^* = 6.1$, the income of each unfaithful member decreases with respect to its equilibrium value, resulting in a total 16,273.03 for members, while the income of the cooperative is 19,380.05.

| Member | $q_j$ | $k_j$ | $k_jq_j$ | $h_j$ | $h_jq_j$ | $(1-h_j)q_j$ | $(k_j-h_j)q_j$ |
|---|---|---|---|---|---|---|---|
| 1 | 60 | 1 | 60 | 0.9 | 54 | 6 | 6 |
| 2 | 90 | 0.9 | 81 | 0.85 | 76.5 | 13.5 | 4.5 |
| 3 | 60 | 1 | 60 | 1 | 60 | 0 | 0 |
| 4 | 100 | 0.9 | 90 | 0.8 | 80 | 20 | 10 |
| 5 | 180 | 0.8 | 144 | 0.75 | 135 | 45 | 9 |
| 6 | 176 | 0.94 | 165.44 | 0.82 | 144.32 | 31.68 | 21.12 |
| Totals | 666 | | 600.44 | | 549.82 | 116.18 | 50.62 |
| Member | Direct | Coop | NonDel | Extra | $F_j$ | $F_{n+1}$ | |
| 1 | 216 | 1,620 | 18 | 219.6 | 1,598.4 | 1,677.6 | |
| 2 | 486 | 2,295 | 13.5 | 494.1 | 2,273.4 | 2,667.6 | |
| 3 | 0 | 1,800 | 0 | 0 | 1,800 | 1,800 | |
| 4 | 720 | 2,400 | 30 | 732 | 2,358 | 2,862 | |
| 5 | 1,620 | 4,050 | 27 | 1,647 | 3,996 | 5,454 | |
| 6 | 1,140.5 | 4,329.6 | 63.36 | 1,159.488 | 4,247.232 | 4,918.848 | |
| Totals | 4,182.5 | 16,494.6 | 151.86 | 4,252.188 | 16,273.03 | 19,380.05 | |

Let us suppose now the cooperative, leaving invariant $p_0^* = 0.1$, changes penalty rate $p_1^*$ from 6.1 to non-equilibrium value $p_1 = 0.6$, which does not satisfy condition (10) of Theorem 1, from Table 4 we can see that some of the members (namely members 2, 4, 5 and 6) benefit from their unfaithfulness.

## 5. Discussion

Our theorems show that an appropriate penalty set by the cooperative can force the members to sell the contracted quantities to the cooperative. This is true even in the case if, at the moment of the corresponding strategy choice, the players have only probabilistic information on the possible market price. From the estimations made in the proofs of both theorems we can see, if inequalities (8) and (9) are strict, then the respective Nash equalities in Theorem 1 and 2 for all members are strict, which means a strong motivation for the members to stick to their equilibrium strategies.

From the particular structure of the payoff functions it is clear that the obtained Nash equilibria are also *attractive solutions*, with the cooperative as distinguished player. In Larbani (1997), apart from the introduction of the concept of an attractive solution, a general existence theorem was proved. In the present paper, instead of

**Table 4.** If the cooperative deviates from its equilibrium strategy pair $(p_0^*, p_1^*) = (0.1, 6.1)$, setting $(p_0^*, p_1) = (0.1, 0.6)$, some of the members are better off by their unfaithfulness. The total income of all members amounts to 20,106.97.

| Member | $q_j$ | $k_j$ | $k_j q_j$ | $h_j$ | $h_j q_j$ | $(1-h_j)q_j$ | $(k_j-h_j)q_j$ |
|---|---|---|---|---|---|---|---|
| 1 | 60 | 1 | 60 | 0.9 | 54 | 6 | 6 |
| 2 | 90 | 0.9 | 81 | 0.85 | 76.5 | 13.5 | 4.5 |
| 3 | 60 | 1 | 60 | 1 | 60 | 0 | 0 |
| 4 | 100 | 0.9 | 90 | 0.8 | 80 | 20 | 10 |
| 5 | 180 | 0.8 | 144 | 0.75 | 135 | 45 | 9 |
| 6 | 176 | 0.9 | 165.44 | 0.82 | 144.32 | 31.68 | 21.12 |
| Totals | 666 | | 600.44 | | 549.8 | 116.2 | 50.62 |
| Member | Direct | Coop | NonDel | Extra | $F_j$ | $F_{n+1}$ | |
| 1 | 216 | 1,620 | 18 | 21.6 | 1,796.4 | 1,479.6 | |
| 2 | 486 | 2,295 | 13.5 | 48.6 | 2,718.9 | 2,222.1 | |
| 3 | 0 | 1,800 | 0 | 0 | 1,800 | 1,800 | |
| 4 | 720 | 2,400 | 30 | 72 | 3,018 | 2,202 | |
| 5 | 1,620 | 4,050 | 27 | 162 | 5,481 | 3,969 | |
| 6 | 1,140.48 | 4,329.6 | 63.36 | 114.048 | 5,292.672 | 3,873.408 | |
| Totals | 4,182.48 | 16,495 | 151.86 | 418.248 | 20,106.97 | 15,546.11 | |

the application of this existence theorem, we have explicitly given this solution. Although among the other refinements of Nash equilibrium (cf. Van Damm 1995) there isn't any with a distinguished player as the cooperative in our case, it is an open question whether some dynamic approach to the strategic choice of players (see e.g. the evolutionary-dynamic approach of Garay 2002) may lead to a dynamically stable equilibrium. (ESS) which is also a particular Nash equilibrium, We note that in Varga et al. (2010), under symmetry conditions, for the "one-shot" version of the game a dynamically stable strict Nash equilibrium was obtained.

While in earlier publications Larbani and Scarelli (2004), Varga et al. (2010), the considered game was based on a single decision of each player, in the present paper the effect of a two-step decision of the players is modelled in terms of a normal form game. The obtained results open the way to modelling of more complex conflicts, including decision mechanisms within the cooperative.

In a more general setting, instead of members of a cooperative the first $n$ players could be independent producers, and player $(n+1)$ a wholesale market operator rather than a cooperative set up by producers. Then, however, producers probably wouldn't accept a threat of a punishment proportional to their "extra profit", which in the case of a cooperative may be acceptable since the members are interested in maintaining the cooperative in the long-term. In our setup, it this "overpunishment" that forces the producers to accept the Nash equilibrium.

Finally, our model can also be extended to the case of an oligopolistic market, where the market price decreases with the increase of the product supply. In the paper Varga et al. (2010), for a one-shot version of the game between a cooperative and its members this extension has been done for a Cournot type oligopoly market with linear inverse demand function, in the present case it may be a lot more complicated.

**Acknowledgment** The authors thank the anonymous referees for their suggestions to improve the paper.

## References

Cobia, D. (1989). *Cooperatives in Agriculture*. Englewood Cliffs, New Jersey, Prentice Hall.

Garay, J. (2002). Many Species Partial Adaptive Dynamics. *BioSystems*, 65, 19–23.

Larbani, M. (1997). About the Existence of the Nash-Slater Equilibrium for a Non-Cooperative Game under Uncertainty. In Caballero, R., Ruiz, F., and Steur, R. (eds.), *Advances in Multi Objective and Goal Programming*. New York, Springer Verlag, 254–262.

Larbani, M. and Scarelli, A. (2004). The Dynamic Control of the Unfaithfulness in a Cooperative. Proceedings of 35th Annual Conference of the Italian Operations Research Society, September 7–10, 2004, Lecce, Italy, p. 42.

Van Damm, E. (1992). Refinements of Nash Equilibrium. In Laffont, J. J. (ed.), *Advances in Economic Theory: Sixth World Congress Vol. I.* Cambridge, Cambridge University Press, 32–75.

Varga, Z., Scarelli, A., Cressman, R. and Garay, J. (2010). Evolutionary Game Model for a Marketing Cooperative with Penalty for Unfaithfulness. Nonlinear Analysis. *Real World Applications*, 11(2), 742–749.